Solution of Nonlinear Brusselator Model by a Combined Sawi Transform and Homotopy Analysis Method

R.A. ODERINU[1],K. A. SALAUDEEN[2],P.I. FARAYOLA [3],W. A. TIJANI [4]

Department of pure and Applied Mathematics, Ladoke Akintola University of Technology, Ogbomoso, Nigeria, [2,3] Emmanuel Alayande University of Education, Oyo,Department of Mathematics and Computing Science Education, [4]Department of pure and Applied Mathematics, Ladoke Akintola University of Technology, Ogbomoso.

E-mail: raoderinu@lautech.edu.ng, salaudeenka@eauedoyo.edu.ng, farayolapi@eauedoyo.edu.ng, adiguntijani111@gmail.com

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INTRODUCTION

- Rationale of Solving Differential Equations
- Approximate Techniques such as ADM (Faiz *et al.* ,2022), VIM (Alawad *et al.,2013)*, *DTM (*Al-Ahmad *et al.*,2020*)*
- Statement of the problem.
- Justification of the study

THEORETICAL FRAMEWORK

Laplace transform of a function f(t) is defined by Laplace 1782 as

$$L\left(f\left(t\right)\right)=\int_{0}^{\infty}e^{-st}f\left(t\right)dt\ ,\left(R\left(s\right)>0\right), \qquad (1)$$

where;

F(s)=Laplace transform of f(t)
s= complex number
t= real number>=0

and

the function f(t) is a piece-wise continuous and of exponential order (Mohmed *et al.,* 2021)

THEORETICAL FRAMEWORK

The basic ideal of Homotopy Analysis Method was introduced by Saed (2020)

consider the following differential equation

$$N\left(u\left(x,t\right)\right)=0,\left(x,t\right)\in\Omega \qquad (2)$$

$$H\left(\phi,s\right)\equiv\left(1-s\right)\left[L\left(\phi\left(x;s\right)\right)-u_0\left(x\right)\right]-shN\left(\phi\left(x;s\right)\right) \qquad (3)$$

$$\left(1-s\right)L\left[\phi\left(t;s\right)-u_0\left(t\right)\right]=sh\left\{N\left[\phi\left(x;t;s\right)\right]\right\} \qquad (4)$$

The zero-order deformation equation was constructed by Liao (2004)

THEORETICAL FRAMEWORK

Sawi transform of a function f(t) as (Higazy *et al.*, 2020)

$$S\left(f\left(t\right)\right)=\frac{1}{w^{2}}\int_{0}^{\infty}e^{-\left(\frac{t}{w}\right)}f\left(t\right)dt\;=F\left(w\right),\;w>0 \qquad (5)$$

Riemann-Liouville fractional order α integral operator

$$I^{\alpha}f\left(t\right)=\frac{1}{\Gamma\left(\alpha\right)}\int_{0}^{x}\left(x-t\right)^{\alpha-1}f\left(t\right)dt=\frac{1}{\Gamma\left(\alpha\right)}\left\{t^{\alpha-1}*f\left(t\right)dt\right\},\;\alpha>0,t>0 \qquad (6)$$

Where I^{α} is α integral.(Jamshed *et al.*,2022; Yuan *et al.*,2023)

REVIEW OF LITERATURE (THEORETICAL FRAMEWORK)

$$ {}_{a}^{c}D_{x}^{\alpha} f(x) = \frac{1}{\Gamma(n-\alpha)} \int_{a}^{x} (x-t)^{n-\alpha-1} f(t) dt = I^{n-\alpha} f^{n}(x) \quad n-1 < \alpha < n \qquad (7) $$

$$ \alpha \in N,\ t > 0 $$

(Lei and Jian, 2018; Wafa *et al.*, 2022)

ESTABLISHMENT OF SOME PROPERTIES OF SAWI TRANSFORM

Theorem 1

If $f(t)$ is a piecewise continuous function of an exponential order and Riemann-Liouville fractional integral of order $\alpha > 0$ of f(t) is given as ${}^{R.L}_{0}I^{\alpha}_{x} f(x)$ then, the Sawi transform of Riemann-Liouville fractional order integral is defined as

$$S\left({}^{R.L}{}_{0}I_{x}^{\alpha} f(x)\right) = F(w)w^{\alpha} \qquad (8)$$

$$\left({}^{R-L}_{0}I^{\alpha}_{x} f(x)\right) = \frac{1}{\Gamma\alpha}\left(x^{\alpha-1} * f(t)\right) \qquad (9)$$

$$S\left({}^{R.L}{}_{0}I_{x}^{\alpha} f(x)\right) = \frac{1}{\Gamma\alpha} S\left(x^{\alpha-1}\right) \cdot S\left(f(t)\right) \qquad (10)$$

ESTABLISHMENT OF SOME PROPERTIES OF SAWI TRANSFORM CONT'D

$$S\left({}^{R-L}_{\quad 0}I_x^{\alpha}\right)=\frac{1}{\Gamma\alpha}\,\Gamma\alpha\bullet w^{\alpha}F(w)=F(w)w^{\alpha} \qquad (11)$$

ESTABLISHMENT OF SOME PROPERTIES OF SAWI TRANSFORM CONT'D

Theorem 2

If f(t) is a piecewise continous function of an exponential order and Caputo fractional derivative is given then, the Sawi transform of Caputo fractional order derivative is given as:

$$S\left[{}_0^cD_x^\alpha f(x)\right] = w^{-\alpha}p(w) - w^2\sum_{m=1}^{n} w^{m-\alpha} f^{m-1}(0) \qquad (12)$$

$$D^\alpha f(x) = \frac{1}{\Gamma(n-\alpha)}\int_0^x (x-t)^{n-\alpha-1} f(t)dt = I^{n-\alpha} f^n(x) \qquad (13)$$

$$S\left[{}_0^cD_x^\alpha f(x)\right] = S\left[{}_0I_x^{n-\alpha} f^n(x)\right] \qquad (14)$$

ESTABLISHMENT OF SOME PROPERTIES OF MOHAND' AND SAWI'S TRANSFORMS CONT'D

$$S\left[I_x^{n-1} f^n(x)\right] = w^{n-\alpha} g(w) \tag{15}$$

$$\frac{1}{\Gamma(n-\alpha)} \int_0^x (x-t)^{n-\alpha-1} f(t)dt = I^{n-\alpha} f^n(x) \tag{16}$$

$$\frac{1}{\Gamma(n-\alpha)} \int_0^x (x-t)^{n-\alpha} f(t)dt = \frac{1}{\Gamma(n-\alpha)} \left(x^{n-\alpha} * f(x)\right) \tag{17}$$

$$S\left(I^{n-\alpha} f^n(x)\right) = \frac{1}{\Gamma(n-\alpha)} S\left(x^{n-\alpha} F(w)\right)$$

$$= \frac{1}{\Gamma(n-\alpha)} * \Gamma(n-\alpha) w^{n-\alpha} g(w) = w^{n-\alpha} g(w) \tag{18}$$

ESTABLISHMENT OF SOME PROPERTIES OF MOHAND' AND SAWI'S TRANSFORMS CONT'D

$$S\left(I^{n-\alpha} f^{n}(x)\right) = w^{n-\alpha} g(w) \tag{19}$$

$$S\left[g(x) = S\left(f^{n}(x)\right)\right] = \frac{F(w)}{w^{n}} - w^{2} n \sum_{m=1}^{n} w^{m-n} F^{m-1}(0) \tag{20}$$

$$S\left(I^{n-\alpha} f^{n}(x)\right) = w^{n-\alpha}\left[\frac{F(w)}{w^{n}} - w^{2} \sum_{m=1}^{n} w^{m-n} F^{m-1}(0)\right] \tag{21}$$

$$S\left[{}_{0}^{c}D_{x}^{\alpha} f(x)\right] = w^{-\alpha} F(w) - w^{2} \sum_{m=1}^{n} w^{m-\alpha} f^{m-1}(0) \tag{22}$$

This end the proof.

PROPOSED SCHEME OF SHAM FOR SOLVING FRACTIONAL ORDER DIFFERENTIAL EQUATION OF CAPUTO TYPE

Consider the fractional order differential equation of Caputo type in an operator form as

$$D_t^{\alpha}\left(y(x,t)\right)+R\left(y(x,t)\right)+F\left(y(x,t)\right)=g(x,t)\ ,\ j-1<\alpha\leq j \qquad (23)$$

$$S\left(D_t^{\alpha}y(x,t)\right)+S\left(\left(Ry(x,t)+Fy(x,t)\right)-g(x,t)\right)=0 \qquad (24)$$

Simplifying eqn. (24) and using Sawi transform of caputo fractional order derivative property to obtain

$$\left[\frac{Y(w)}{w^{\alpha}}-\sum_{k+0}^{n-1}\frac{y^{k}(0)}{w^{\alpha-k+1}}\right]+S[Ry]+S[Fy]=S[g(x,t)] \qquad (25)$$

PROPOSED METHODOLOGY CONT'D

Isolating $F(w)$ in eqn. (25) and using $Y(w)=\bar{y}_n(x,w)$, gives

$$\bar{y}(x,w)-\sum_{k=0}^{n-1}\frac{y^k(0)}{w^{-k+1}}+w^{\alpha}S[R(y)]+w^{\alpha}S[F(y)]-w^{\alpha}S[g(x,t)]=0 \qquad (26)$$

$$S[\bar{y}_n(x,w)-\chi_n\bar{y}_{n-1}]=hD_{n-1}\left[\begin{array}{l}\bar{\phi}(x,w,q)-\sum_{k=0}^{n-1}\frac{y^k(0)}{w^{-k+1}}+w^{\alpha}S[R(y)]\\ +w^{\alpha}S[F(y)]-w^{\alpha}S[g(x,t)]\end{array}\right] \qquad (27)$$

$$\bar{y}_n(x,w)=\chi_n\bar{y}_{n-1}$$
$$-\left[\bar{y}_{n-1}(x,w)+(1-\bar{\chi}_{n-1})\left[\begin{array}{l}-\sum_{k=0}^{n-1}\frac{Y^k(0)}{w^{-k+1}}\\ -w^{\alpha}S[g(x,t)]+D_{n-1}[w^{\alpha}S(R(y)+F(y))]\end{array}\right]\right] \qquad (28)$$

PROPOSED METHODOLOGY CONT'D

Simplifying Eqn. (28) gives

$$\bar{y}_n = -(1-\chi_n)\bar{y}_{n-1} - (1-\bar{\chi}_{n-1})\left[\sum_{k=0}^{n-1}\frac{Y^k(0)}{w^{-k+1}} + S[g(x,t)]\right] - w^{\alpha}S[R(y)+F(y)] \qquad (29)$$

initial approximation is obtained from Eqn. (29) as

$$y_0(x,t) = S^{-1}\left[-(1-\bar{\chi}_{n-1})\left[\sum_{k=0}^{n-1}\frac{Y^k(0)}{w^{-k+1}} + S[g(x,t)]\right]\right] \qquad (30)$$

Applying Sawi inverse on Eqn. (29), leads to

$$y_n(x,t) = S^{-1}\begin{bmatrix} \chi_n\bar{y}_{n-1} - \bar{y}_{n-1} \\ -(1-\bar{\chi}_{n-1})\begin{bmatrix} -\sum_{k=0}^{n-1}\frac{y^k(0)}{w^{-k+1}} \\ -w^{\alpha}S[g(x,t)] + D_{n-1}[w^{\alpha}S(R(y)+F(y))] \end{bmatrix} \end{bmatrix} \qquad (31)$$

Numerical Applications

(1) Consider nonlinear fractional order Brusselator model

$$\left.\begin{aligned}
&{}^{C}D_t^{\psi} p(x,y,t) - p^2(x,y,t)q + 2p(x,y,t) - \frac{1}{4}\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}\right) = 0 \\
&{}^{C}D_t^{\psi} q(x,y,t) - p(x,y,t) + p^2(x,y,t)q - \frac{1}{4}\left(\frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial y^2}\right) = 0 \\
&p(x,y,0) = e^{-x-y} \\
&q(x,y,0) = e^{x+y}
\end{aligned}\right\} \qquad (32)$$

(Faiz *et al.*, *2022*)

Numerical Applications Cont'd

(2) Consider nonlinear fractional order Brusselator model

$$\left.\begin{aligned}
&\frac{\partial^{\beta} p}{\partial t}(x,y,t)-1-p^{2}q(x,y,t)-\frac{3}{2}p(x,y,t)-\frac{1}{4}\left(\frac{\partial^{2} p}{\partial x^{2}}+\frac{\partial^{2} p}{\partial z^{2}}\right)=0\\
&\frac{\partial^{\beta} q}{\partial t}(x,y,t)-\frac{1}{2}p(x,y,t)+p^{2}q(x,y,t)-\frac{1}{4}\left(\frac{\partial^{2} q}{\partial x^{2}}+\frac{\partial^{2} q}{\partial z^{2}}\right)=0\\
&p(x,y,0)=x^{2}\\
&q(x,y,0)=y^{2}
\end{aligned}\right\} \qquad (33)$$

(Faiz *et al., 2022*)

RESULTS AND DISCUSSION CONT'D

Table 1a. Comparison of error of SHAM and LADM with the exact, for x=1=y, at different values of t at ψ=1 of p(x,y,t) for Equation (32)

t	SHAM	LADM (Faiz *et al.,2022*)
0.1	0.0000027847	0.0000056040
0.2	0.0000220106	0.0000445589
0.3	0.0000734128	0.0001494809
0.4	0.0001720181	0.0003522209
0.5	0.0003322108	0.0006839010
0.6	0.0005677973	0.0011749511
0.7	0.00089206811	0.00185514134
0.8	0.00131785958	0.00275361562
0.9	0.00185761345	0.00389892128
1.0	0.00252343554	0.00531903845
MAE	**0.000776121098**	**0.00162374335**

RESULTS AND DISCUSSION CONT'D

Table 1b. Comparison of error of SHAM and LADM with the exact, for x=1=y, at different values of t at ψ =1 of q(x,y,t) for Equation (32)

t	SHAM	LADM(Faiz *et al.,2022*)
0.1	0.000001943	0.000007582
0.2	0.000031414	0.000076526
0.3	0.000160658	0.000312910
0.4	0.000512983	0.000873877
0.5	0.001265375	0.001970246
0.6	0.002651249	0.003869267
0.7	0.004963340	0.006897510
0.8	0.008556750	0.011443900
0.9	0.013852160	0.017962970
1.0	0.021339130	0.026978100
MAE	**0.0053335002**	**0.0070392888**

RESULTS AND DISCUSSION CONT’D

Table 1c. SHAM of p(x,y,t) at x=1 and y=1 for different values of t at different values of ψ forEquation (32)

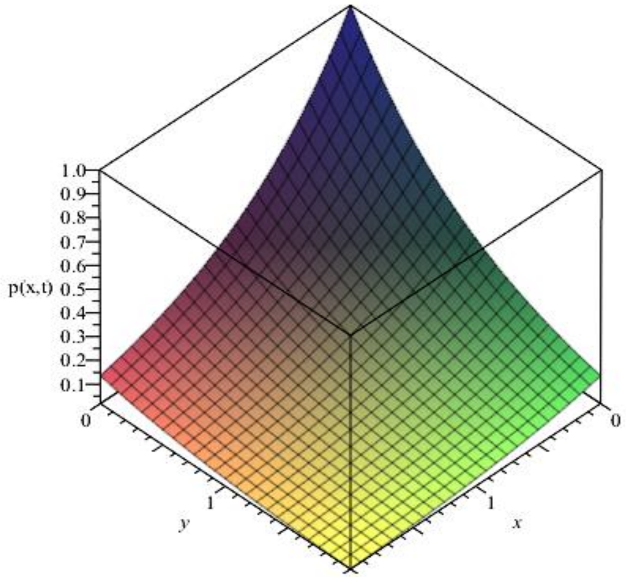
t	ψ =0.7	ψ =0.8	ψ =0.9	ψ =1
0.1	0.1215606283	0.1244150615	0.1267976259	0.1287376883
0.2	0.1140583819	0.1170891746	0.1199203966	0.1224784389
0.3	0.1083227223	0.1110531889	0.1138383638	0.1165575706
0.4	0.1036740549	0.1058928949	0.1083705797	0.1109751765
0.5	0.09981401855	0.1014158879	0.1034284350	0.1057314354
0.6	0.09657496242	0.09750952672	0.09895804192	0.1008266410
0.7	0.09384961828	0.09409934832	0.09492272196	0.09626123033
0.8	0.09156351517	0.09113225071	0.09129573140	0.09203581287
0.9	0.08966195166	0.08856835130	0.08805666516	0.08815119995
1.0	0.08810304998	0.08637655163	0.08518946440	0.08460843416

RESULTS AND DISCUSSION CONT'D

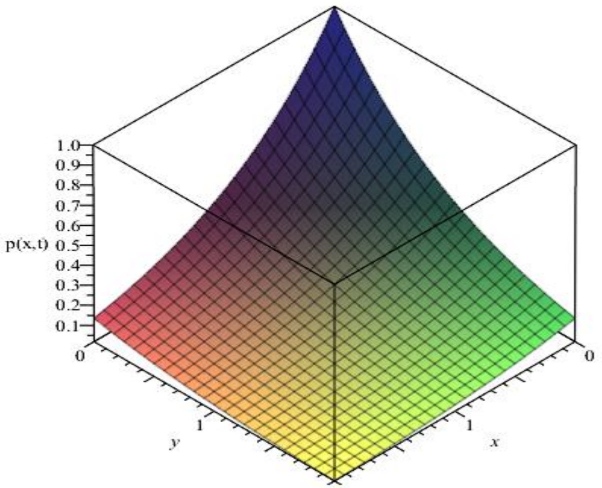
Table 1d. SHAM of q(x,y,t) at x=1 and y=1 for different values of t at ψ =0.7, 0.8, 0.9 and 1 for Equation (32)

t	ψ =0.7	ψ =0.8	ψ =0.9	ψ =1
0.1	8.262811881	8.051411450	7.890562004	7.767899163
0.2	8.877266883	8.588470154	8.355155038	8.166138499
0.3	9.448068854	9.108300777	8.823611425	8.584697739
0.4	10.00258420	9.627042637	9.303347521	9.024500516
0.5	10.55174673	10.15151938	9.797885999	9.486470461
0.6	11.10122620	10.68546443	10.30940649	9.971531206
0.7	11.65439739	11.23122970	10.83947027	10.48060638
0.8	12.21345241	11.79043987	11.38930440	11.01461963
0.9	12.77990439	12.36429361	11.95993518	11.57449456
1.0	13.35484584	12.95372067	12.55225869	12.16115483

RESULTS AND DISCUSSION CONT'D



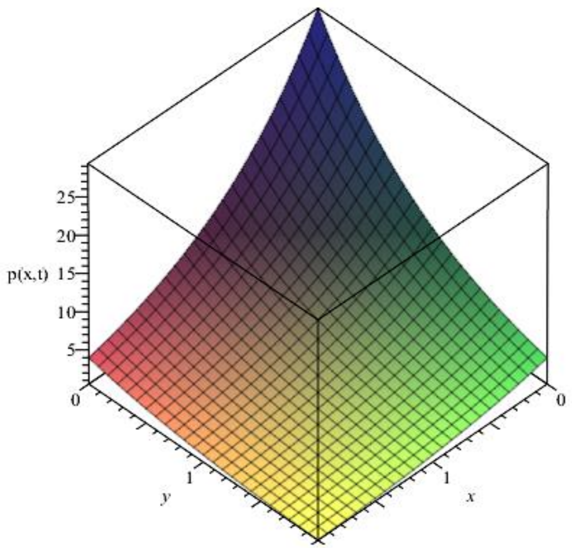
SHAM



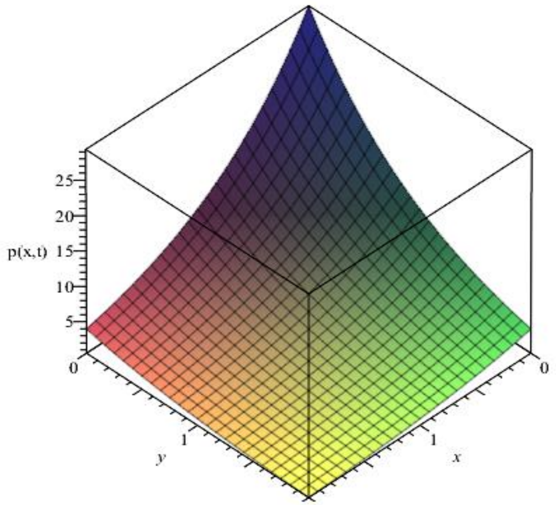
Exact

Figure 1a: Solution of Exact and SHAM of p(x,y,t) at t=0,ψ=1 for Equation (32)

RESULTS AND DISCUSSION CONT'D



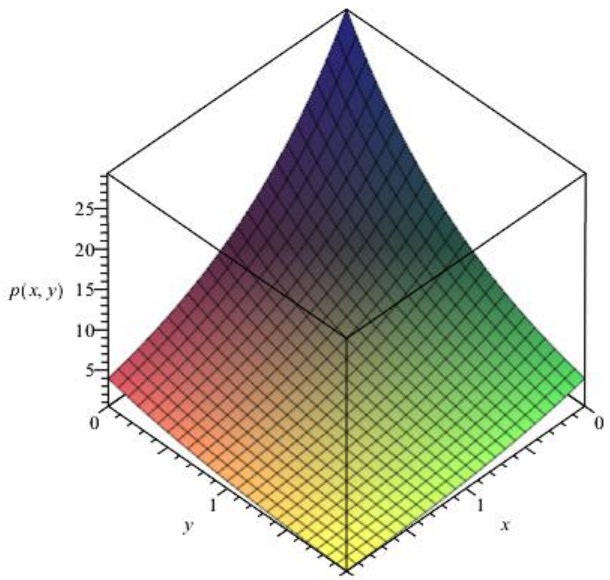
SHAM



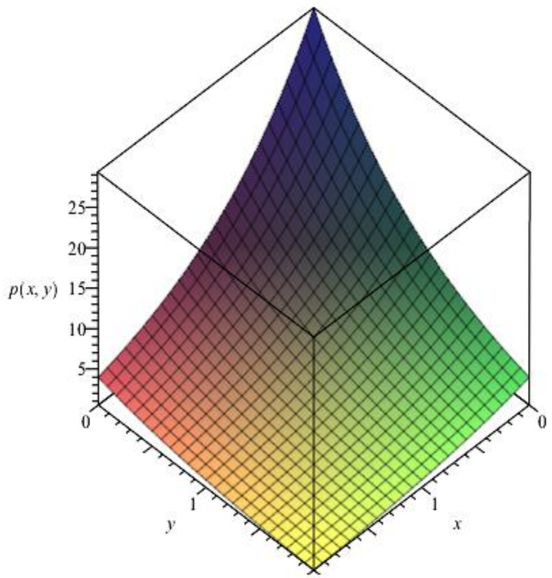
Exact

Figure 1b: Solution of Exact and SHAM of p(x,y,t) at t=5, ψ=1 for Equation (32).

RESULTS AND DISCUSSION CONT'D



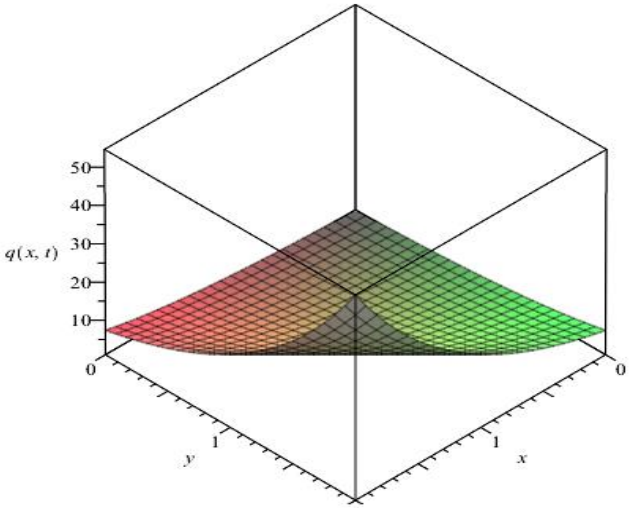
SHAM



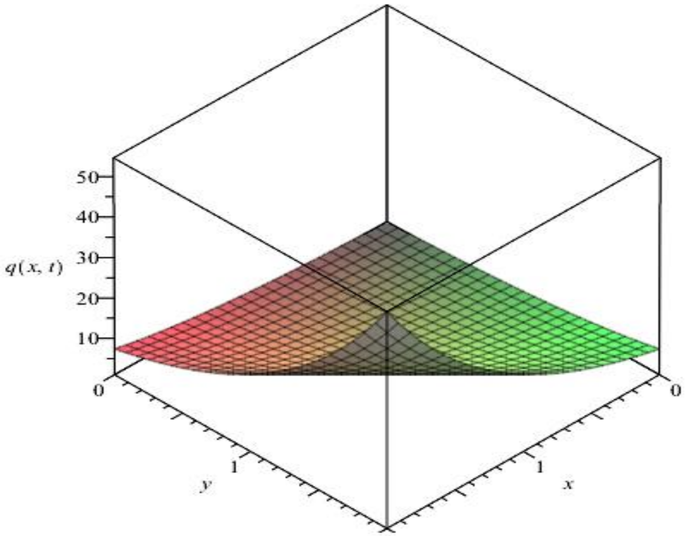
Exact

Figure 1c: Solution of Exact and SHAM of p(x,y,t) at t=10, ψ=1 for Equation (32)

RESULTS AND DISCUSSION CONT'D



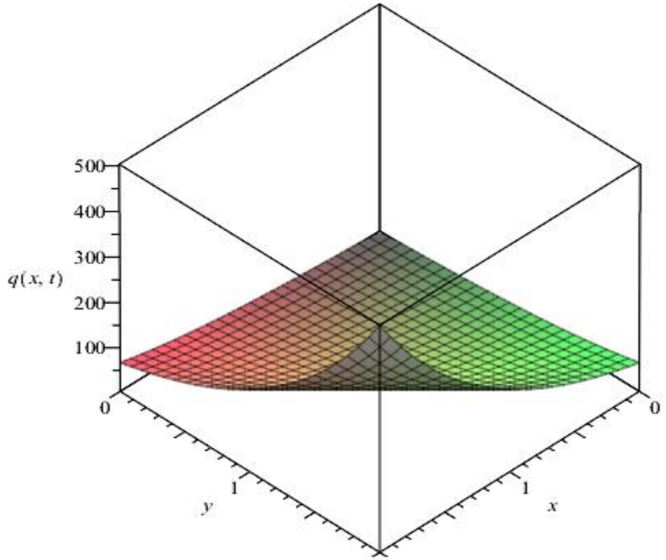
SHAM



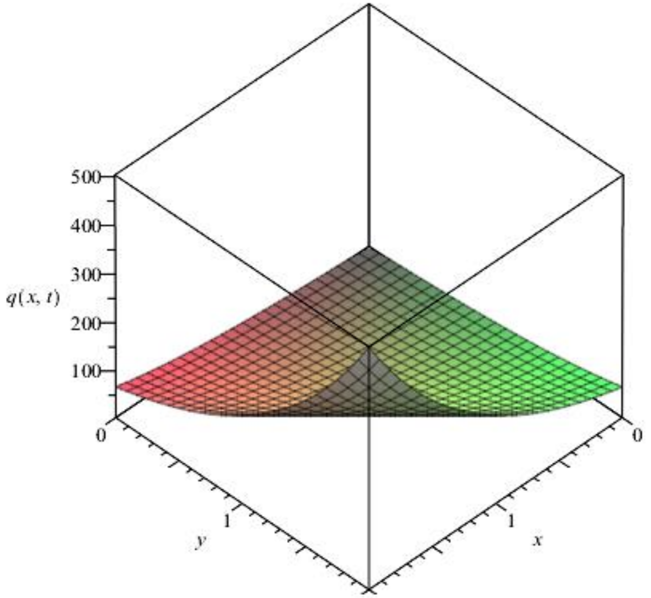
Exact

Figure 1d:Solution of Exact and SHAM of q(x,y,t) at t=0, =1 for Equation (32)

RESULTS AND DISCUSSION CONT'D

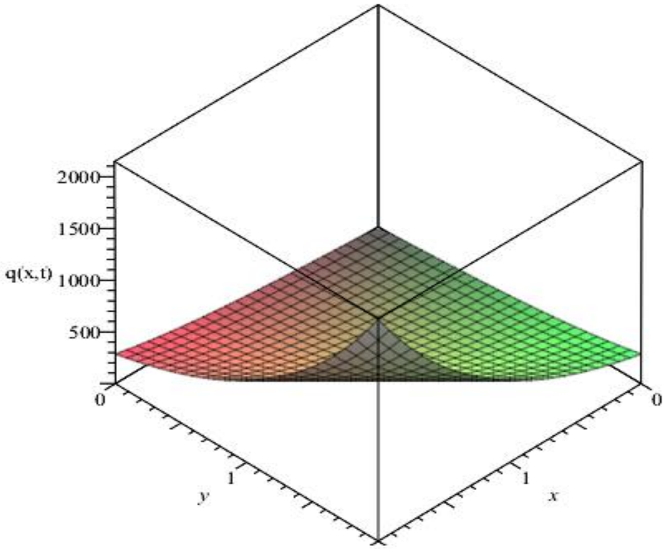


SHAM

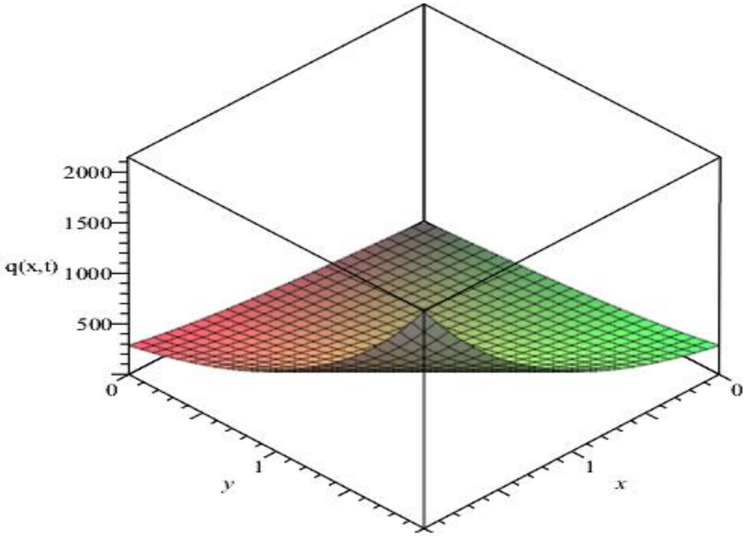


Exact

Figure 1e: Solution of Exact and SHAM of q(x,y,t) at t=5, ψ=1 for Equation (32)



SHAM



Exact

Figure 1f: Solution of SHAM and Exact of q(x,y,t) at t=10, ψ=1 for Equation (32)

RESULTS AND DISCUSSION CONT'D

Table 2a. Comparison of SHAM, LADM for x=1=y, at different values of t at ψ=1 of p (x,y,t) for Equation (33).

t	SHAM	LADM **(Faiz *et al*., 2022)**
0.1	1.121250000	1.121250000
0.2	1.285000000	1.285000000
0.3	1.446250000	1.446250000
0.4	1.740000000	1.740000000
0.5	2.031250000	2.031250000
0.6	2.365000000	2.365000000
0.7	2.741250000	2.741250000
0.8	3.160000000	3.160000000
0.9	3.621250000	3.216250000
1.0	4.125000000	4.125000000

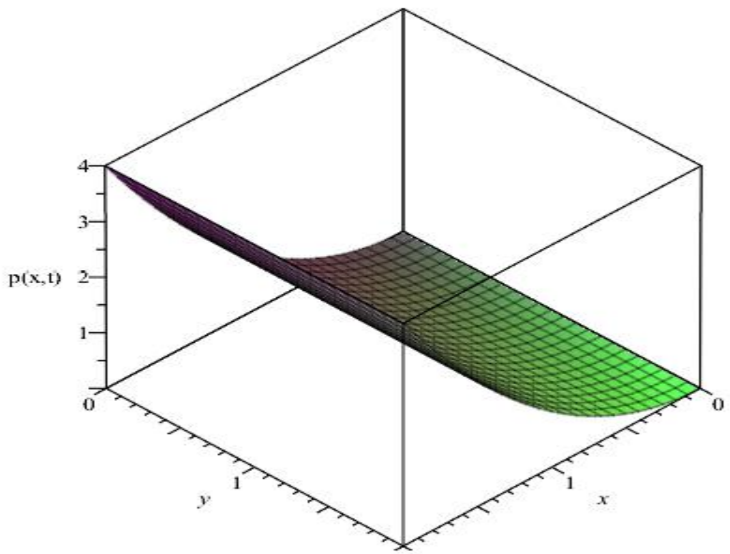
RESULTS AND DISCUSSION CONT'D

Table 2b. Comparison of SHAM and LADM for x=1=y, at different values of t at ψ=1 of q (x,y,t) for Equation (33).

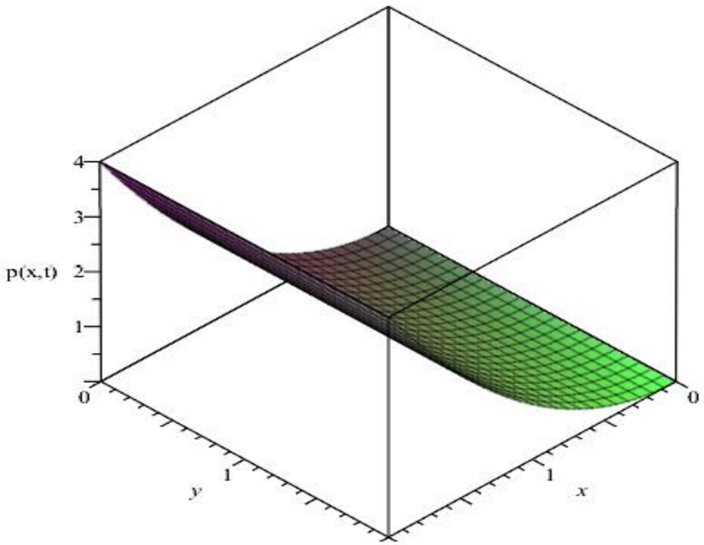
t	SHAM	LADM **(Faiz *et al*., 2022)**
0.1	0.9762500000	0.9762500000
0.2	0.9050000000	0.9050000000
0.3	0.7862500000	0.7862500000
0.4	0.6200000000	0.6200000000
0.5	0.4062500000	1.906250000
0.6	0.1450000000	0.1450000000
0.7	-0.163750000	-0.163750000
0.8	-0.520000000	-0.520000000
0.9	-0.923750000	-0.923750000
1.0	-1.375000000	-1.375000000

Table 2c. SHAM of p(x,y,t) at x=1 and y=1 for different values of t at ψ =0.7, 0.8, 0.9 and 1 for Equation (33)

t	ψ =0.7	ψ =0.8	ψ =0.9	ψ =1
0.1	1.355797803	1.244839002	1.171075239	1.121250000
0.2	1.716181507	1.522643163	1.384170723	1.285000000
0.3	2.107927833	1.842869417	1.642108187	1.491250000
0.4	2.528116046	2.202064400	1.942997897	1.740000000
0.5	2.973947312	2.597323287	2.285194445	2.031250000
0.6	3.443165331	3.026328624	2.667336293	2.365000000
0.7	3.933952241	3.487197803	3.088282011	2.741250000
0.8	4.444816923	3.978366383	3.547056973	3.160000000
0.9	4.974512998	4.498508112	4.042815054	3.621250000
1.0	5.521981058	5.046479745	4.574811308	4.125000000

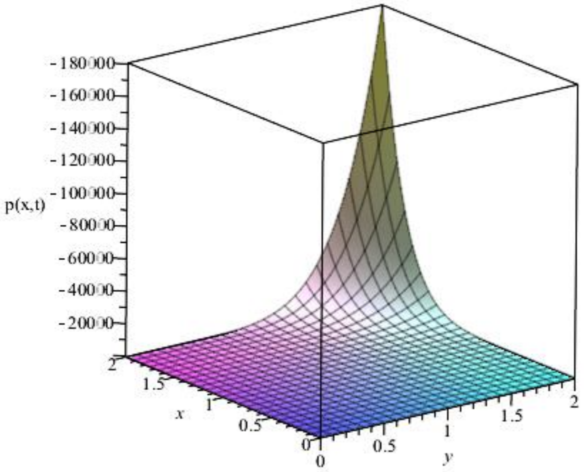


SHAM

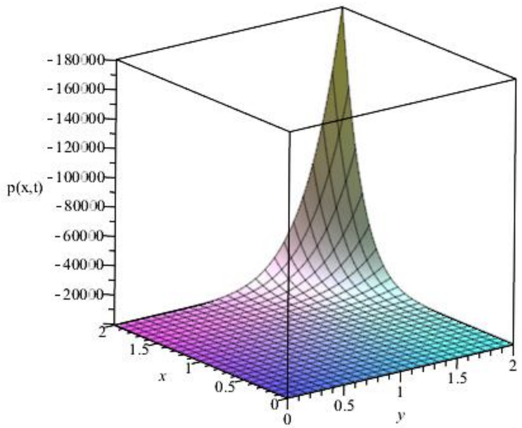


LADM (Faiz *et al.*, 2022)

Figure 2a: Graph of SHAM and LADM of p(x,y,t) at t=0,ψ=1 f or Equation (33)

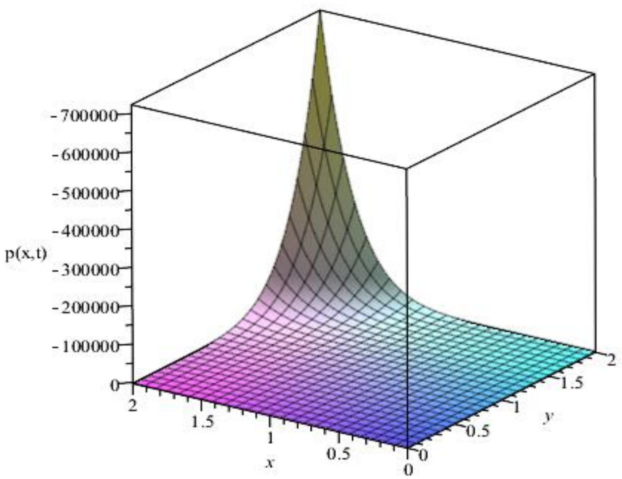


SHAM

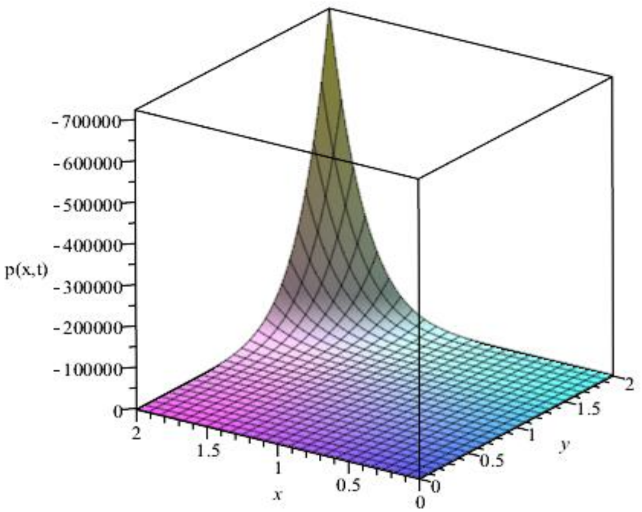


LADM (Faiz *et al., 2022)*

Figure 2b: Solution of SHAM and LADM of p(x,y,t) at t=5,ψ=1 f or Equation (33)



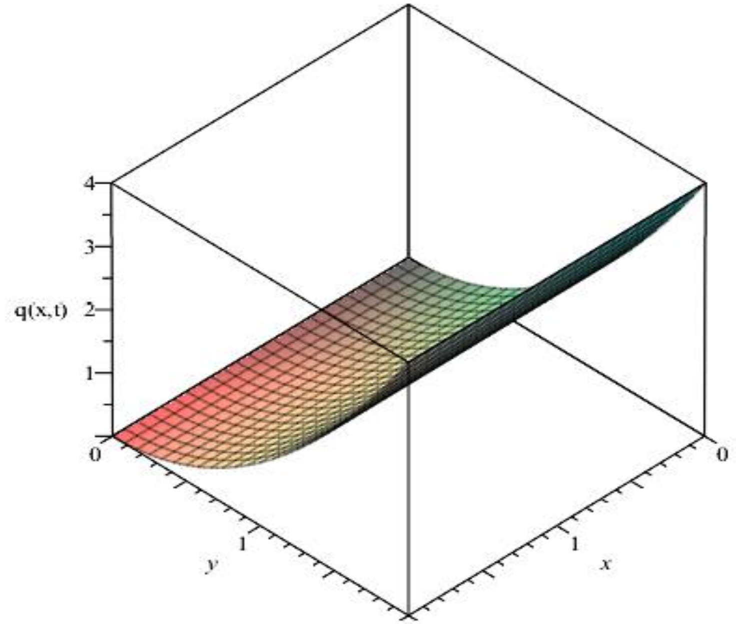
SHAM



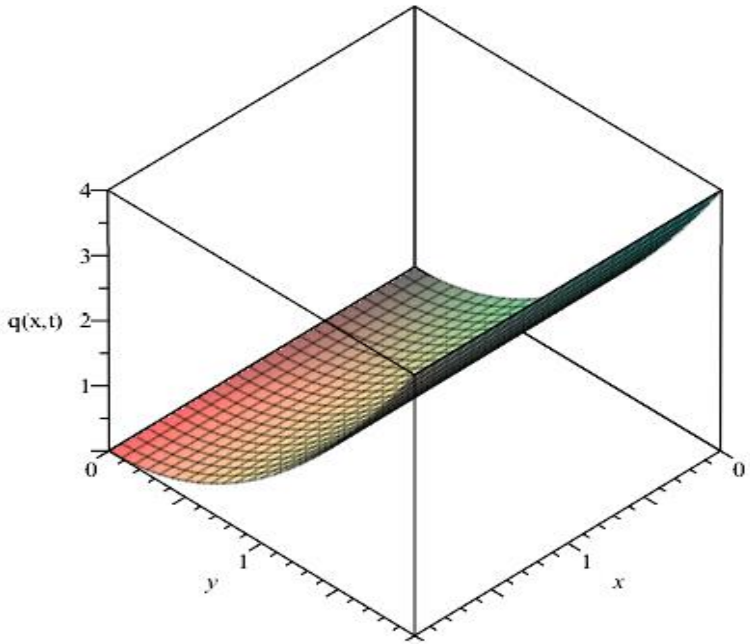
LADM (Faiz *et al., 2022)*

Figure 2c:Solution of SHAM and LADM of p(x,y,t) at t=10,ψ=1 f or Equation (33)

RESULTS AND DISCUSSION CONT'D

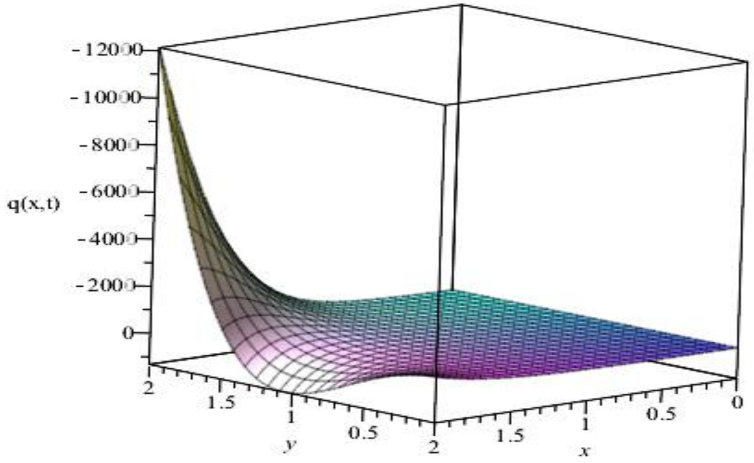


SHAM

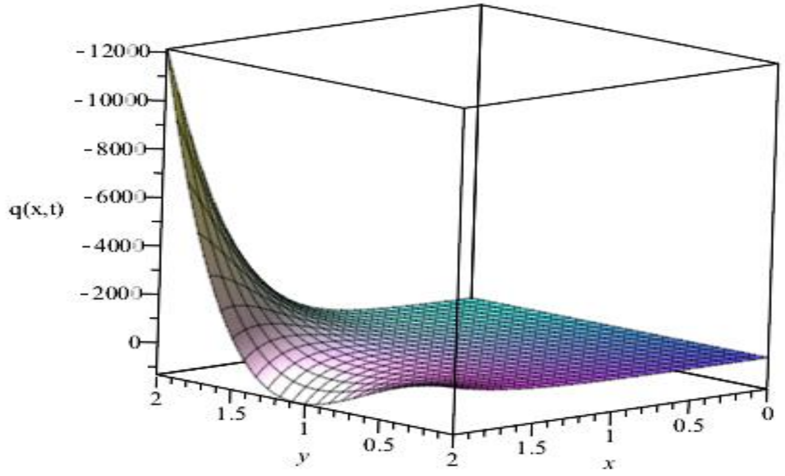


LADM (Faiz *et al., 2022)*

Figure 2d :Solution of SHAM and LADM of q(x,y,t) at t=0,ψ=1for Equation (33)



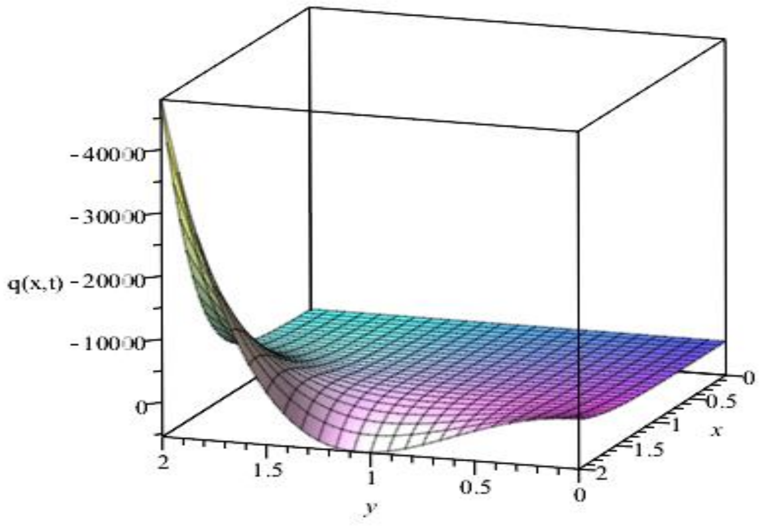
SHAM



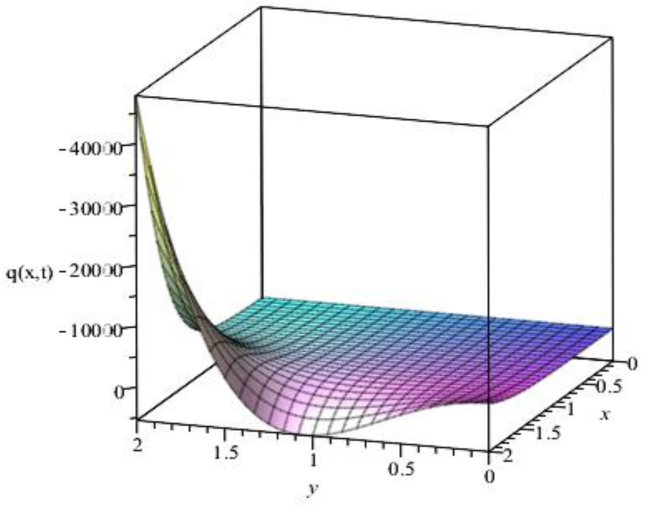
LADM (Faiz *et al., 2022)*

Figure 2e: Solution of SHAM and LADM of p(x,z,t) at t=5,ψ=1 f or Equation (33)

RESULTS AND DISCUSSION CONT'D



SHAM



, **LADM (**Faiz *et al., 2022)*

Figure 2f: Solution of SHAM and LADM of q(x,y,t) at t=10, ψ=1 f or Equation (33)

CONCLUSION

- Sawi transform of the nth order derivative, Riemann Liouville fractional order integral and Caputo properties were established by using principle of mathematical induction, definition of Sawi transform, method of integration by parts, definition of convolution of two functions, definition of Caputo fractional order derivative;
- New schemes for solving fractional order differential equations of Caputo types were derived by combining Sawi transform and HAM;
- To justify the effectiveness of the proposed schemes, some identified problems were solved and results were compared with existing solutions;
- The results then revealed that schemes were reliable with wide applicability to solve fractional order differential equations.

CONTRIBUTIONS TO KNOWLEDGE

- ✓ New integral schemes for solving non integer order partial differential differential equations were established and applied.
- ✓ Sawi transform of the nth derivative of a function f(t), Riemann-Liouville integral and fractional derivative in Caputo sense were established.

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THANK YOU EVERYONE FOR YOUR AUDIENCE