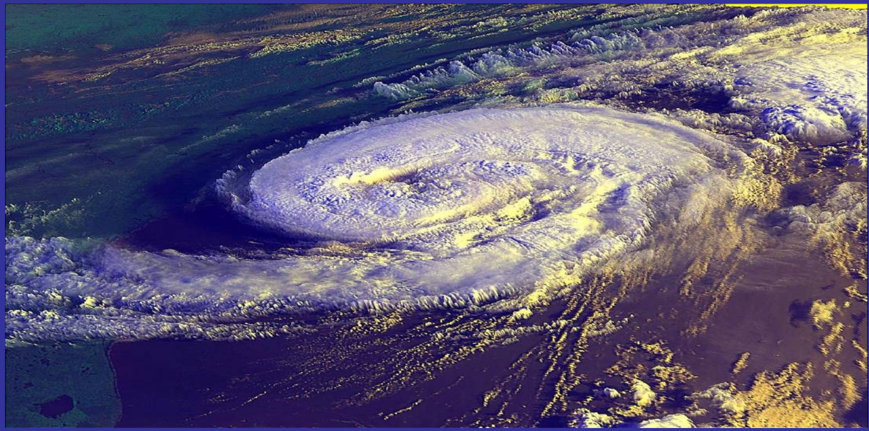
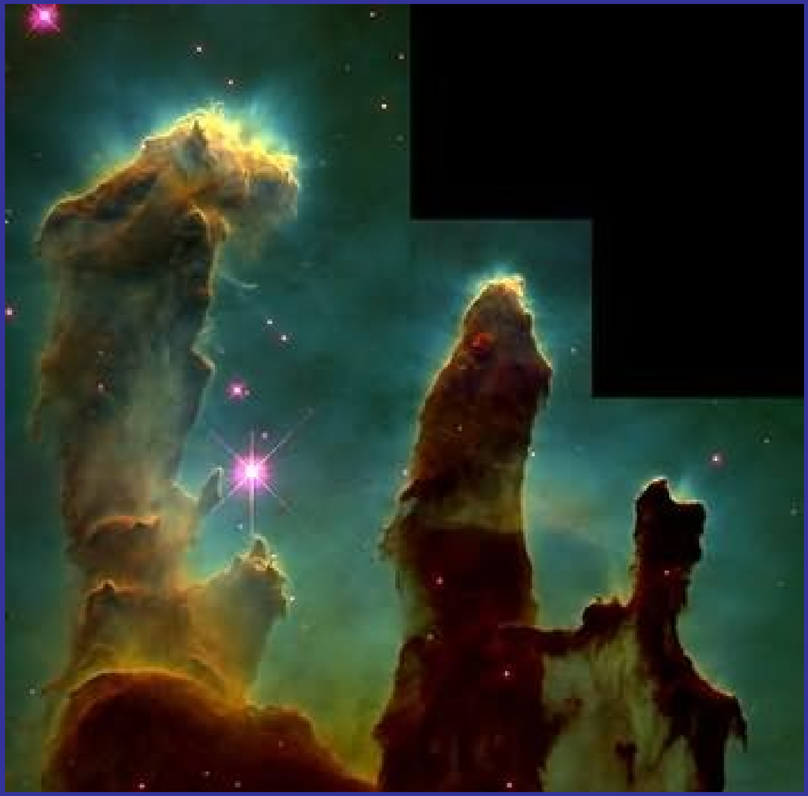
"Perhaps the fundamental equation that describes the swirling nebulae and the condensing, revolving, and exploding stars is just a simple equation for the hydrodynamic behavior of nearly pure hydrogen gas".
-- Richard Feynman





Fluids and fluid instabilities, including turbulence, appear in a wide range of natural contexts as well as engineering systems.

The vortices produced by the flapping wings of a fruit fly (Drosophila) are smaller than 1 mm in size, while the length scale associated with colliding galaxies is if order light years.

The time scale of the eddies produced by the hovering Drosophila is less than one second, while the time scale for colliding galaxies is billions of years.

Flows on these vastly different scales are *described by the same basic equations.*

Hydrodynamic (continuum) approximation

Fluids can be considered as continuous fields if the characteristic macroscopic motions are much larger than molecular motions, i.e. if

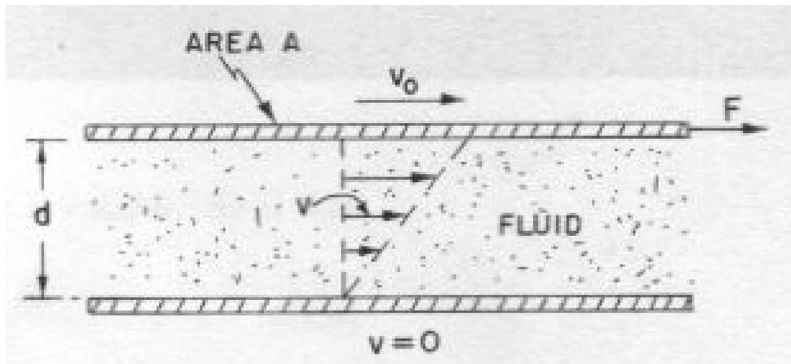
$$L_{hydro} >> mfp$$

There needs to be a sufficient number of atoms so that a density can be assigned and further that relative fluctuations in density will be negligibly small (they decrease as $N^{-1/2}$, where N is the number of atoms in a volume with the characteristic dimension of the length scale we are considering).

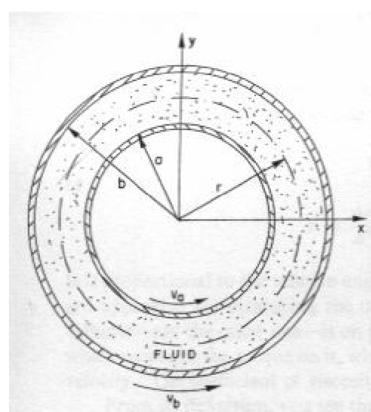
In this case, we can use continuum mechanics and express the equations of motion in terms of partial differential equations

Viscous forces

If you apply a shearing force to a fluid it will move— the shear forces are described by the viscosity. Consider a layer of fluid between two plates, one stationary and one moving at a slow speed v_0



Shear stress: $$\frac{F}{A} = \eta \frac{v_0}{d}$$ where F is the force required to keep the upper plate moving and η is the dynamic or shear viscosity.

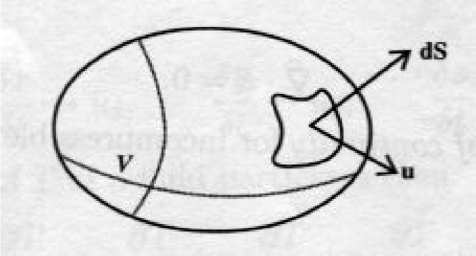


A more practical viscometer due to Mallock and Couette

Basic Fluid Equations

The equations of mass, momentum and energy conservation are written down in a coordinate system that is fixed in space ("Eulerian" description).

Considering a fluid of density ρ, moving at a velocity **u:** the mass flux through an element of surface area **d*s*** of a volume V of fluid must be equal to the rate of mass loss in the volume:



$$\int \rho \vec{u} \cdot d\vec{s} = -\frac{d}{dt} \int_V \rho dv$$

Mass conservation then takes the form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0$$

Assuming constant density (note: we will assume this even in the case where there is thermal expansion due to heating) this reduces to the divergenceless condition (one of the advantages of dealing with incommmpressible fluids):

$$\nabla \cdot \mathrm{u} = 0$$

Like the magnetic field **B** in electrodynamics the fluid velocity has zero divergence (there is a close analogy to the equations of electrodynamics).

For the conservation of momentum we write down the equations in terms of a force per unit volume *f.*

$$\rho \cdot (\mathrm{acceleration}) = \mathrm{f}$$

$$\mathrm{f} = -\nabla \mathrm{p} + \mathrm{f}_{\mathrm{visc}} + \mathrm{f}_{\mathrm{ext}}$$

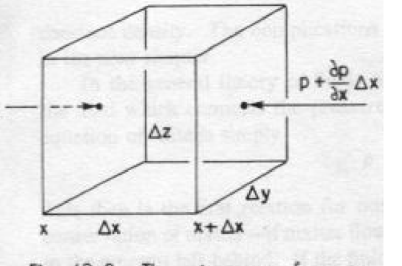


Fig. 40–3. The net pressure force on a cube is $-\nabla p$ per unit volume.

Note: the acceleration is *not* simply $\frac{\partial v}{\partial t}$

A fluid particle can also change its momentum by flowing to a place where the velocity is different.

In fact, the rate of change of any quantity, say "G", is given by

$$\frac{\mathrm{DG}}{\mathrm{Dt}} = \frac{\partial G}{\partial t} + \mathrm{u}_x \frac{\partial G}{\partial x} + \mathrm{u}_y \frac{\partial G}{\partial y} + \mathrm{u}_z \frac{\partial G}{\partial z}$$

$$\frac{\partial G}{\partial t} + \vec{u} \bullet \nabla G$$

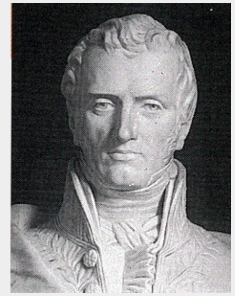
where the operator

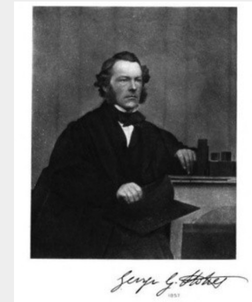
$$\frac{D}{\mathrm{Dt}} \equiv \frac{\partial}{\partial t} + \vec{u} \bullet \nabla$$

is the *substantial* or *convective* derivative

Consider simple pressure-driven flows:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\,\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v}$$





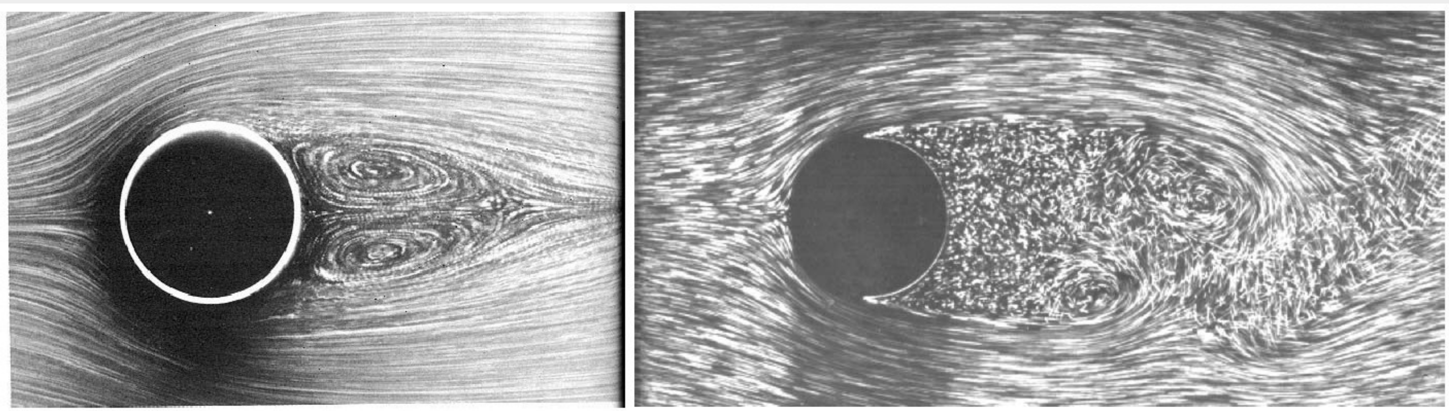
(Navier - Stokes)

$$\mathbf{v}' = \frac{\mathbf{v}}{U} \qquad \mathbf{r}' = \frac{\mathbf{r}}{L} \qquad t' = \frac{tU}{L} \qquad p' = \frac{p}{\rho U^2}$$

$$\frac{\partial \mathbf{v}'}{\partial t} + (\mathbf{v}' \cdot \nabla')\,\mathbf{v}' = -\nabla p' + \frac{1}{Re}\nabla'^2 \mathbf{v}'$$

Re = UL/ν

Reynolds



a

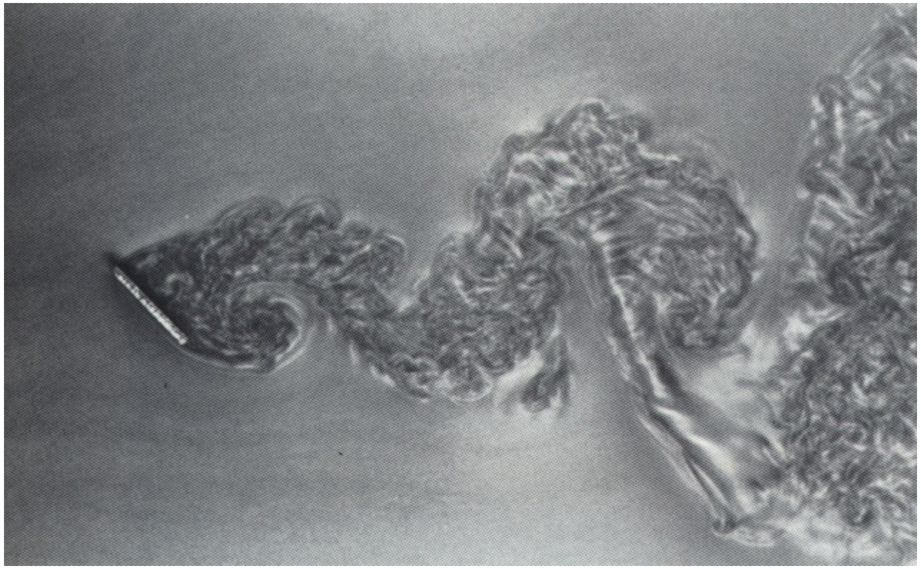
b

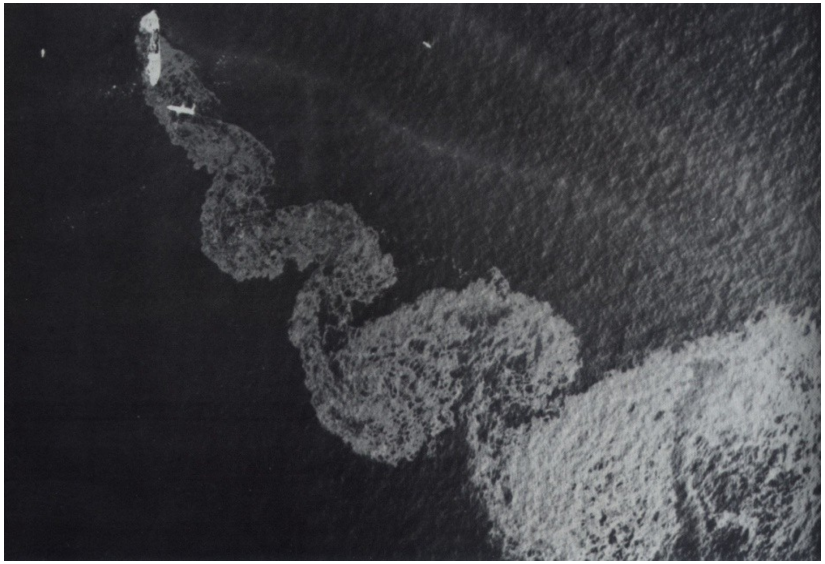
Flow past a circular cylinder. (a) Re = 26. (b) Re = 2000.

Dynamical similarity

If we consider two simple flows that are *geometrically similar*, then they are also *dynamically similar* if the corresponding *Re* is the same for both, *regardless of the specific velocities, lengths and fluid viscosities involved.*

Matching such parameters between laboratory testing of a model and the actual full-scale object is the principle upon which aerodynamic model-testing is based. We will touch on these applications later.





Above left: wake behind a flat plate in the laboratory inclined 45 degrees to the direction of the flow (left to right). Above right: A foundered ship in the sea inclined 45 degrees to the direction of the current.

Some dimensional considerations

Turbulent flows are characterized by having high rates of diffusion of momentum and/or heat.

The *molecular* time scale for momentum diffusion is (dimensionally):

$$t_M \sim \frac{L^2}{v}$$

A corresponding time scale for diffusion in turbulent flows can be estimated using the large scale L and velocity u, which are the most effective at mixing :

$$t_T \sim L/u$$

Then

$$\frac{t_M}{t_T} \sim \frac{\mathrm{uL}}{v} = \mathrm{Re}_L$$

Re_L of a turbulent flow then can be interpreted as the ratio of a characteristic molecular time scale to a turbulent time scale in the case that the former is evaluated over the *same length scale*.

It will be useful to coinsider a new vector field **Ω** as the curl of **u**

$$\Omega = \nabla \times u$$

This rotation of the fluid is called the vorticity.

By Stokes theorem we can relate the sum of vorticity over a given area to a circulation around any reducible loop in the fluid bounding that area:

$$\oint u \cdot \mathrm{dl} = \int_A (\nabla \times u) \cdot \mathrm{dA}$$

Note: we already saw in the case of superfluids having zero curl of the velocity there was still a non-zero circulation. This is because of the hollow vortex core which means that the circulation cannot be reduced to a point.

Vortex lines have to end on solid boundaries or on themselves. The latter are vortex rings (e.g., smoke rings).

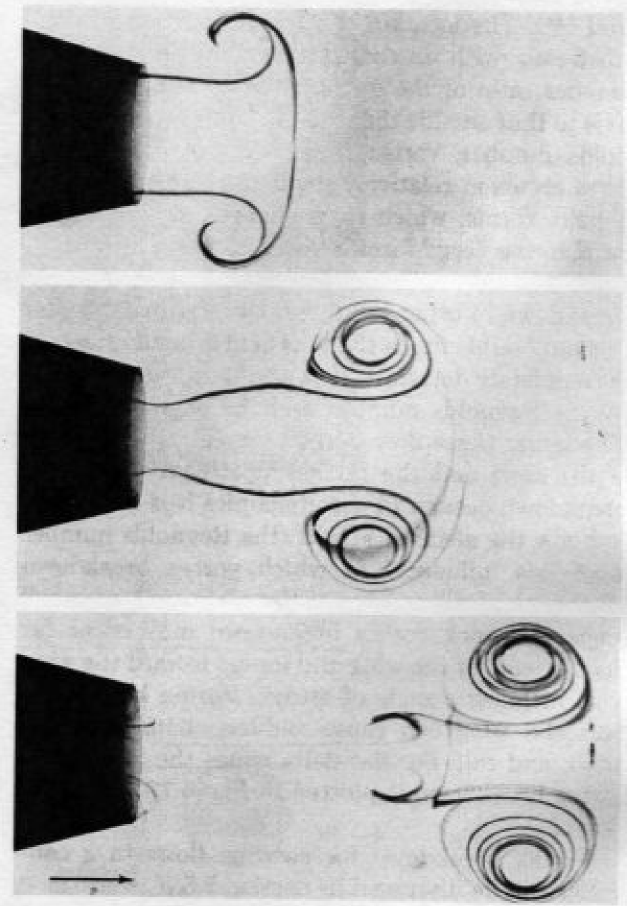


Figure 5.79. Development of a vortex ring in water at a nozzle, made visible by dyed streak lines; Re = 1350 (Photograph by M. Gühler, Fachhochschule Mannheim).

Focusing on turbulence

Besides its impotance in the motion of submarines, ships and aircraft, pollutant dispersion in the earth's atmosphere and oceans, heat and mass transport in engineering applications as well as geophysics and astrophysics turbulence is also a paradigm for strongly nonlinear systems, distinguished by strong fluctuations and strong coupling among a large number of degrees of freedom. [G. Falkovich, K.R. Sreenivasan, *Phy. Today* 59, 43 (2006)].

Turbulence is particularly useful because the equations of motion are known and can be simulated with precision. And so, even distant areas such as fracture [M.P. Marder, *Condensed Matter Physics.* Wiley, New York (2000)]--- perhaps even market fluctuations [B.B. Mandelbrot, *Scientific American* 280, 50 (1999)]---may benefit from a better understanding of it.

The complexity of the underlying equations (Navier -Stokes) has precluded much analytical progress, and the demands of computing power are such that routine fully - resolved simulations of large turbulent flows has not yet been possible.

Thus, the progress in the field has depended heavily on experimental input. This experimental input in turn points in part to a search for optimal test fluids, and the development and utilization of novel instrumentation.

Some defining characteristics of turbulence*

*see Tennekes and Lumley-- a great reference

Irregularity: Turbulent flows are irregular and random. This complexity will exist in both space and time (spatial irregularity itself clearly does not constitute turbulence nor does the converse). Even though the deterministic Navier Stokes equations presumably contain all of turbulence it is impossible to predict the precise values of any variables at any time.

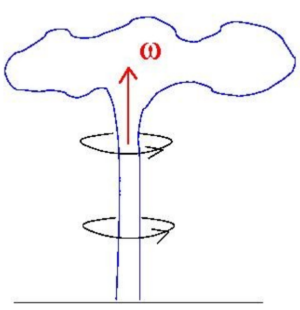
Statistical measures, however, are reproducible and this has led to statistical approaches toward solving the NS equations. This always leads to a situation in which there are more unknowns than equations—the so-called closure problem.

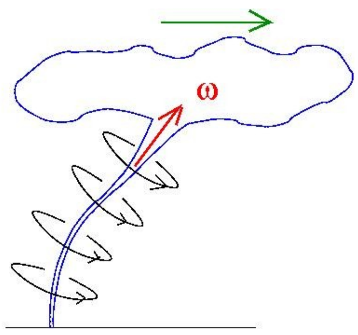
Diffusivity: The most important aspect of turbulence as far as applications are concerned is its associated strong mixing and high rates of momentum, heat and mass transfer. The randomness or irregularity is not sufficient to define turbulence—turbulent flows will always exhibit strong spreading of fluctuations.

High Re: Turbulent flows exist only for high values of Re. They often originate from instabilities in the fluid such as those in RB convection that we looked at yesterday.

Dissipation: If no energy is supplied turbulence will decay rapidly. It needs to acquire energy from its environment. We will look at decaying turbulence in the quantum context today.

Stretching: Turbulence must then be maintained and vortex stretching is an important process.



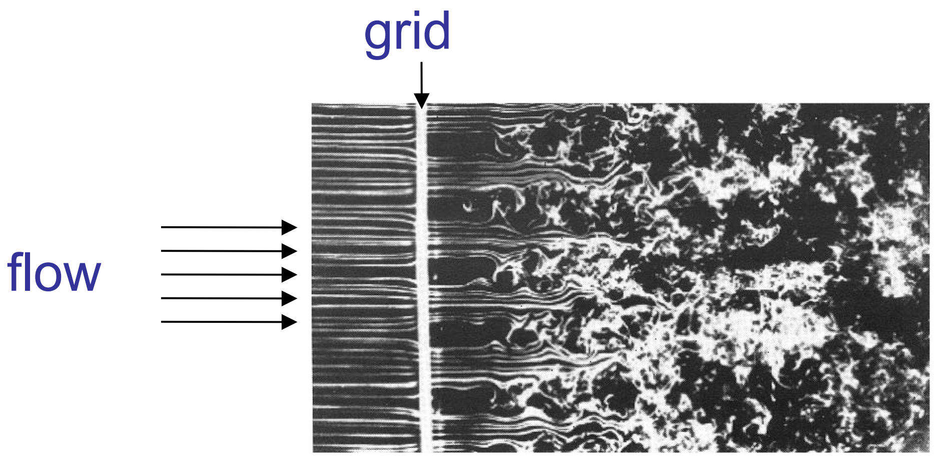


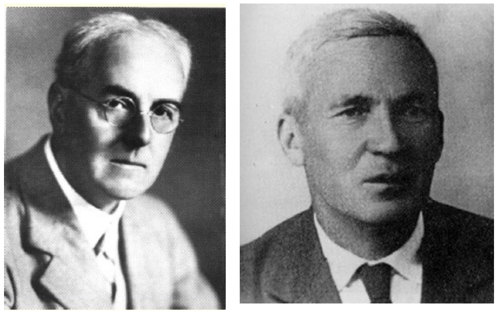
We will have to consider how vortex stretching can occur in quantum turbulence where we have a restriction of a single quantum of vorticity for each vortex filament

Flows: Turbulence is a property of the flow not the fluid, although it is tempting to find effective viscosities or diffusivities that represent the enhanced transport of turbulent flows. Even if it is not a general solution to the problem, it is possible to find situations where it works as we saw yesterday in the context of turbulent thermal convection.

Mathematically, the details of the transition to turbulence remain poorly understood.

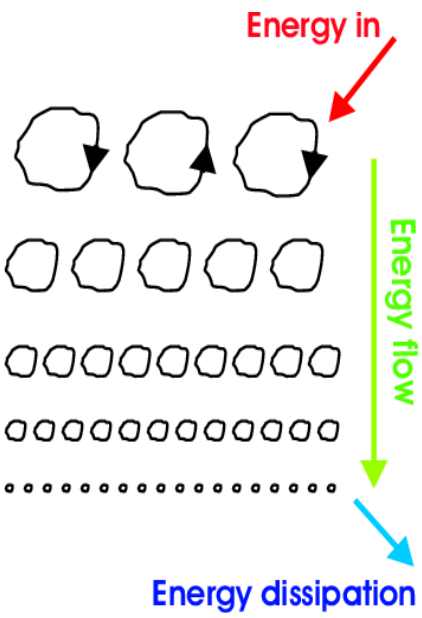
(Approximately) homogeneous isotropic turbulence



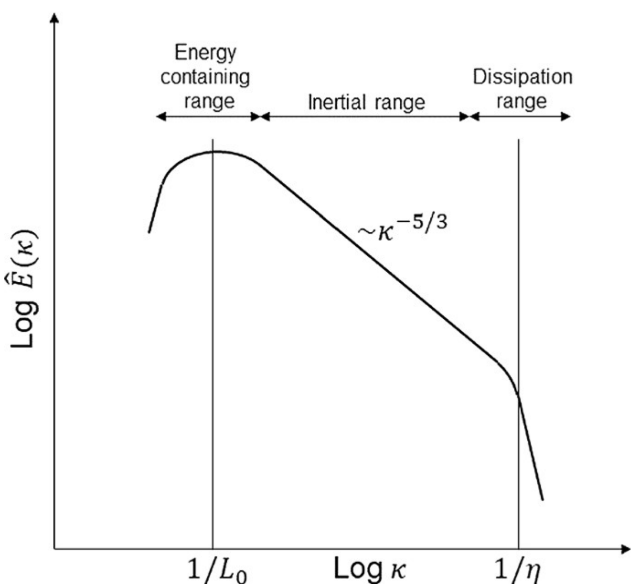


Richardson Kolmogorov

Spatial representation



Wavenumber representation



More on small scales and the energy cascade…

Reducing viscosity (or increasing Re) does not alter the rate of energy dissipation per unit mass, ε, (which is determined from the energy input) but rather allows the cascade of energy to *continue to smaller scales*.

The smallest scale in turbulent flows is the dissipation length scale η, where viscosity becomes dominant. To see this we consider that the rate of energy supplied at the injection scale L per unit mass is given by

$$u^2 \cdot u/\mathrm{L} = \mathrm{u}^3/L$$

This energy is dissipated at a rate per unit mass, ε, which must be the same:

$$\varepsilon = \mathrm{u}^3/L$$

This must be true at *all* scales ℓ (ε is a constant) and so we have in general

$$u_\ell \sim \varepsilon^{1/3} \ell^{1/3}$$

We can now write down the eddy turn-over time for the scale ℓ in terms of the energy dissipation rate:

$$t_\ell \equiv \frac{\ell}{u_\ell} \sim \varepsilon^{-1/3} \ell^{2/3}$$

We can also write down the viscous diffusion time t_ℓ^{diff} for the same scale ℓ:

$$t_\ell^{\mathrm{diff}} \sim \frac{\ell^2}{\nu}$$

Note that the diffusion time goes to zero faster with ℓ than does the eddy turnover time. Viscosity becomes important at a dissipation length scale ℓ_{diss} for which the two time scales are equal, i.e., for

$$\frac{\ell_{\mathrm{diss}}^2}{\nu} = \varepsilon^{-1/3} \ell_{\mathrm{diss}}^{2/3}$$

Denoting this dissipation scale ℓ_{diss} as η (customary notation), we find

$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4} = L\,\mathrm{Re}_L^{-3/4}$$

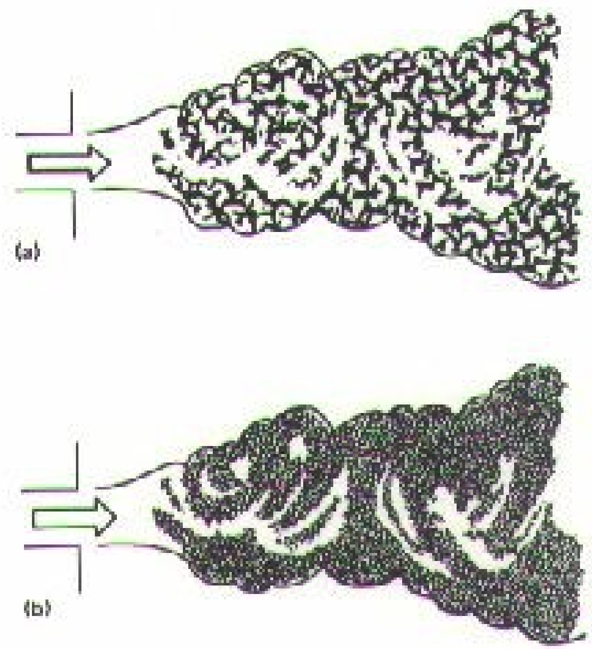
The separation between the energy-injection scales and the dissipative scales increases with Re.

So if there are any *universal statistical properties* of turbulence, it is reasonable to look for them as Re→∞

But… recall from Tuesday that Re→∞ is not the same as Re=∞ because in the former there exists a scale at which viscosity acts (its wavenumber may also go to infinity) while for he latter we have an ideal frictionless fluid at all scales not just an approximation for the large ones.

The difference between two flows with the same integral scale but different Re is the size of the smallest eddies. Index of refraction gradients are steep for the smallest eddies and hence shimmering seen on hot days.

A turbulent jet



(a) "Low" Re. (b) "High" Re

"inner scales" of length, time, and velocity:

$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$$

$$\tau = \left(\frac{\nu}{\varepsilon}\right)^{1/2}$$

$$u_\eta = (\nu\varepsilon)^{1/4}$$

Note that $\frac{\eta u_\eta}{\nu} = 1$

Why an energy *cascade* (local interaction) and *why we can't recover the Euler equations as Re* $\rightarrow\infty$

Physically, distortions of one eddy due to another are caused by shear. The shear associated with scales of any size ℓ is proportional to the gradient in the scale velocity:

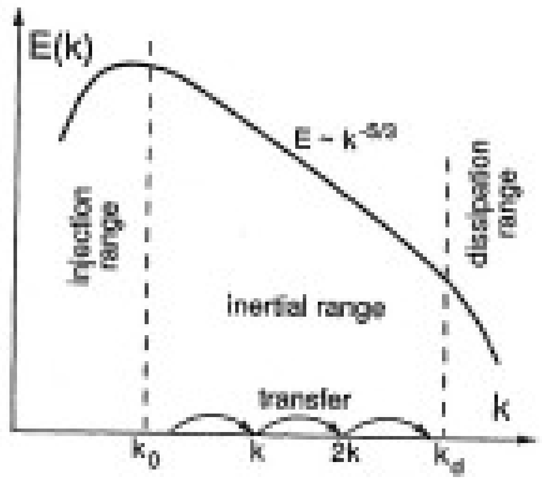
$$s_\ell \sim \frac{u_\ell}{\ell} \sim \varepsilon^{1/3}\,\ell^{-2/3}$$

The greatest shear is at the smallest scales. *Note this latter fact is what keeps the dissipation term in the NS equations from going to zero as Re becomes large.*

So…BIG eddies don't significantly distort much smaller ones because the big ones have little shear* and tiny eddies don't significantly distort much larger ones because the much smaller eddy does not act coherently over the scale of the much larger ones.

*even though eddies of size ℓ will be swept along by eddies of size L >> ℓ (as noted by Leonardo da Vinci) Galilean invariance of the NS equations of motion precludes any consequent change in their energy.

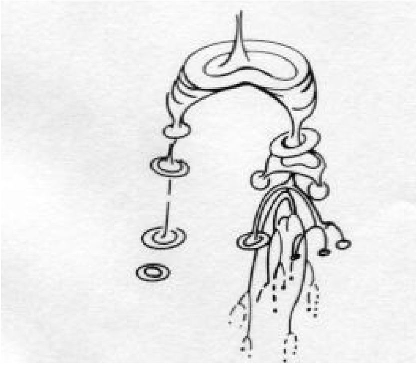
Hence: *a cascade* of energy from *scale to scale*



Dimensional analysis gives:

$$E(k) = C\, \varepsilon^{2/3}\, k^{-5/3}$$

C is a constant of order 1 (from measurements it is estimated to be roughly 1.5)



An example of a “cascade” with a drop of dense ink in water

Q: As Re$\rightarrow\infty$ can we neglect viscosity and recover the simpler Euler equations?

A: No, turbulence will always create small enough scales for viscosity to dominate.

Q: Will the continuum approximation remain valid as Re increase more and more?

A: good question

Molecular and turbulent scales

Let us consider gases: on a molecular level the characteristic length is the mean free path ξ and the velocity scale is the speed of sound *a*. The kinematic viscosity is approximated by:

$$\nu \sim a\xi$$

$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4} = \left(\frac{\nu^4 \mathrm{Lu}}{u^4 \nu}\right) = \left(\frac{\nu}{u}\right)\left(\frac{\mathrm{uL}}{\nu}\right)^{1/4} = \left(\frac{\nu}{u}\right)\mathrm{Re}^{1/4}$$

Mean free path/ turbulent dissipation scale: $$\frac{\xi}{\eta} = \frac{\nu}{a} \bullet \frac{u}{\nu} \bullet \mathrm{Re}^{-1/4} = \frac{M}{\mathrm{Re}^{1/4}}$$

M = Mach number. High M and low Re is an unlikely combination.

Within the Kolmogorov framework, the fluid is incompressible (M << 1) so the higher the Re the better the continuum approximation! Of course if compressibility becomes important then the hydrodynamic approximation may come into doubt. This (compressibility + high Re) can happen in some astrophysical systems.

- Between experiments and theory:

- Direct numerical simulations (DNS): The range of scales needing to be well resolved grows as $Re^{3/4}$, and thus $Re^{9/4}$ in 3 dimensions. The state of the art in DNS is about $Re \sim 10^4$, or about 3-4 orders of magnitude lower than the Re corresponding to a typically commercial jet aircraft, and the same amount for most atmospheric and oceanic flows.

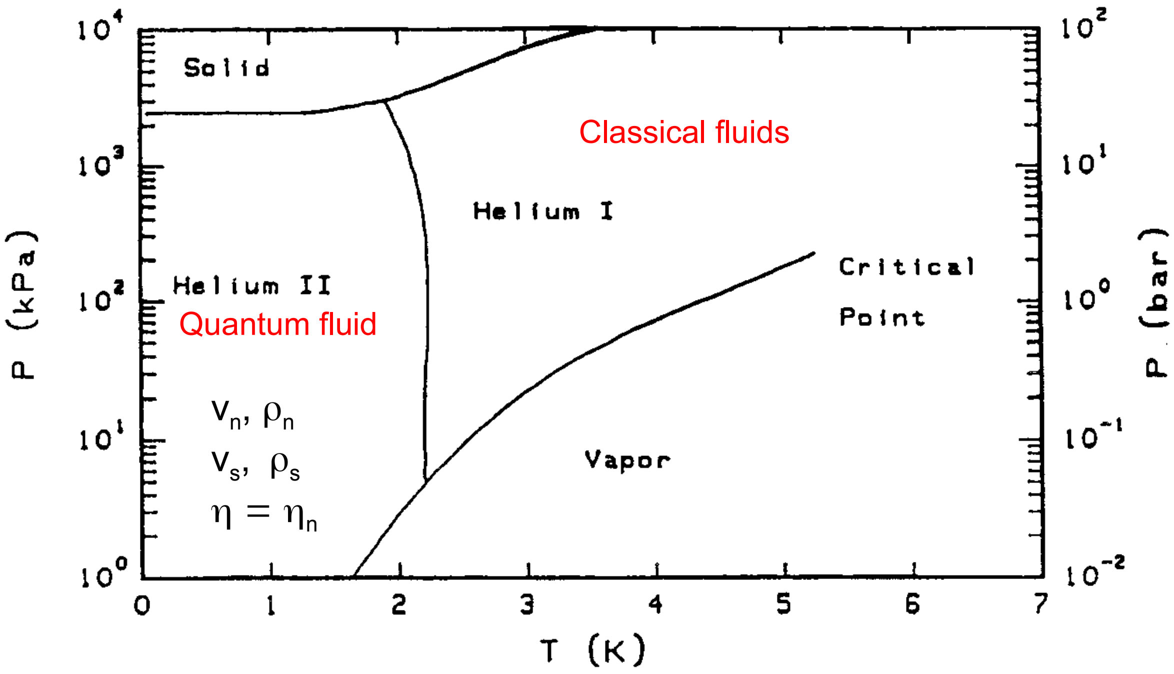
- Large eddy simulations (LES)--which compute *only the large scales* and model the small scales-- do better, but they are not satisfactory for every problem.

Alternatives to laboratory experiments and simulations

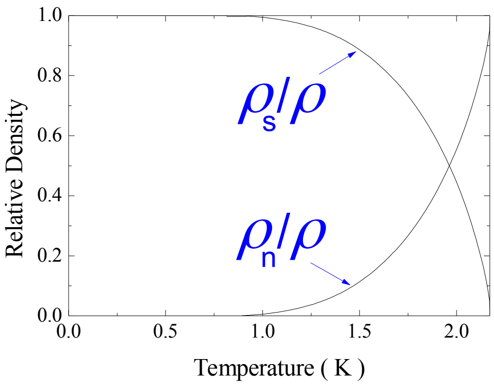
Could nature's turbulence "laboratories", such as the atmosphere and oceans, be instrumented and studied?

Yes…and they are. But this is not a substitute for controlled laboratory experiments, especially when questions become more refined. Boundary conditions, stationarity, need to be considered.

Superfluid turbulence



Below T_λ: two fluid model



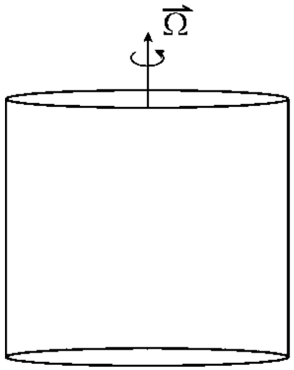
Superfluid irrotational (curl v_s=0)!

Condensate wave function: $\Psi = |\Psi| e^{i\phi} \longrightarrow$ superfluid velocity: $\mathbf{v_s} = \frac{\hbar}{m_4} \nabla \varphi$

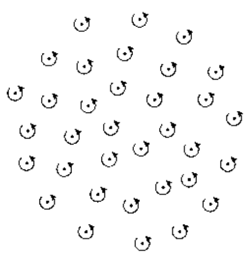
Circulation: $\kappa = \oint \vec{v} \cdot d\vec{l} = \frac{\hbar}{m_4} \triangle \varphi = n \frac{h}{m_4}$ (in a multiply-connected region, otherwise Stokes' theorem doesn't allow it)

Feynmann (PLTP, 1955) envisioned turbulence as a tangle of such "quantized" vortices

A curious observation



Rotating containers of helium II were observed to have a parabolic meniscus (Osborne, 1950). The shape of the meniscus was independent of temperature which was surprising since it was assumed that the superfluid component would not rotate as a solid body.



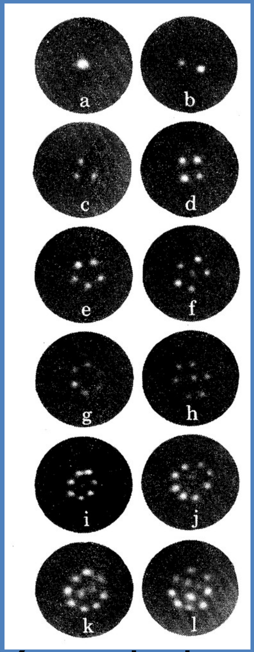
In fact, this was resolved by considering the fluid to be threaded with an array of quantized vortices whose number obeyed Feynman's rule:

$$n = \frac{2\Omega}{\kappa}$$

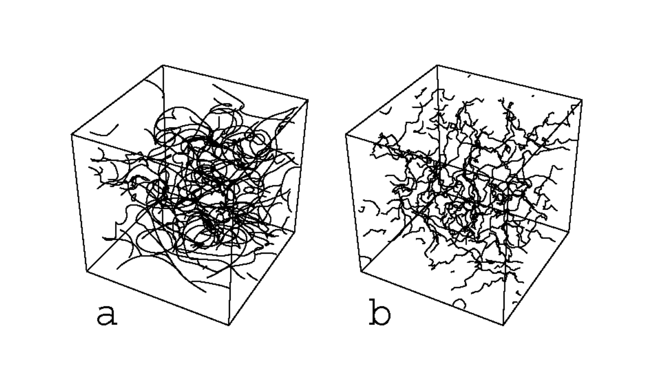
Note, here the angular velocity is denoted by Ω, rather than the vorticity as we used before. The vorticity is equal to 2Ω in solid body rotation, hence Feynman's rule says that a sufficient number of vortices will be produced to mimic solid body rotation in the superfluid. Clearly this only works well for n large.

Regular arrays and irregular tangles of quantized vortices

Visualizing indirectly the regular array of vortices in a rotating bucket



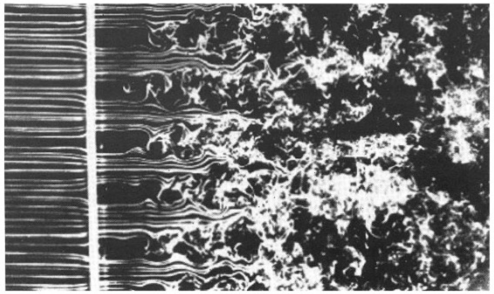
Yarmchuk, et al. 1978



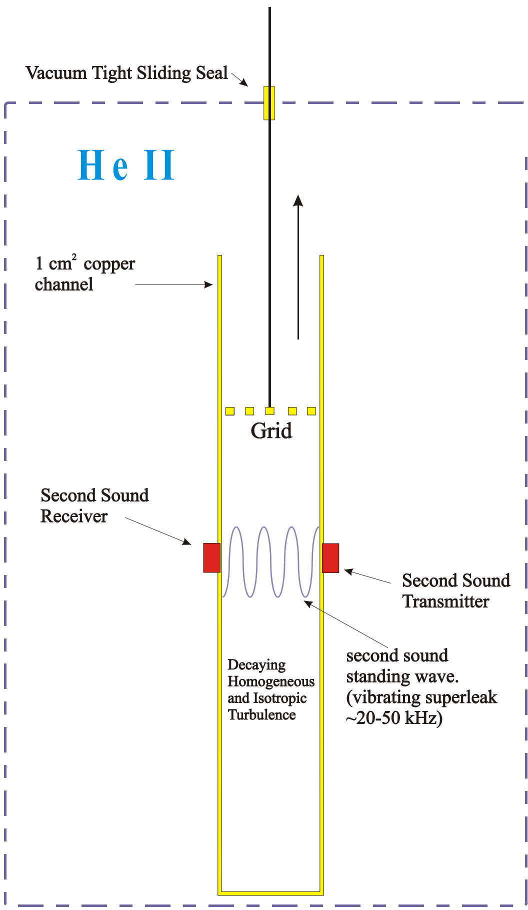
The simulated tangle of quantized vortices on the left corresponds to 1.6K, while that on the right is at 0K. After Tsubota, et al (2000).

As we shall see later, turbulent flows in the Kolmogov sense can mimic eddies on all scales through partial polarization of vortex bundles.

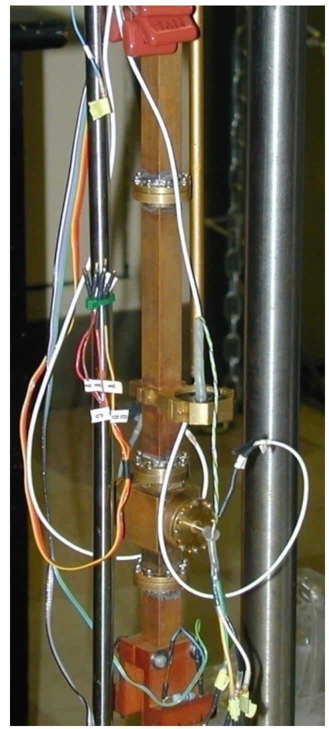
Superfluid grid flow



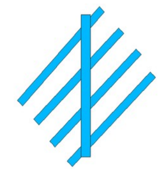
Theses: M.R. Smith, S.R. Stalp



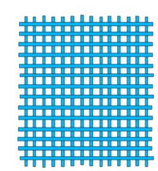
Pocket-size! 1-cm square channel



Original grid: robust, 65% open brass monoplanar grid with tines 1.5 mm thick and mesh spacing of 0.167 cm



Newer grid: 28 rectangular tines of width 0.012 cm forming 13 full meshes across the channel of approximate dimension 0.064 cm.



Measure decay of L = length of vortex line per unit volume

Exciting second sound

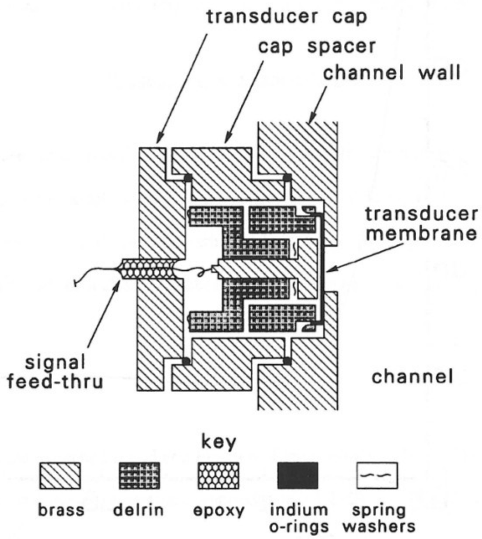


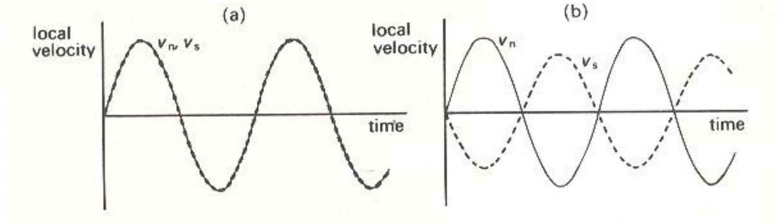
FIGURE 8 Cross section of a typical second sound transducer. The nuclepore membrane is gold plated on the channel side, and is in electrical contact with the channel wall.

Second sound is excited and detected using vibrating nuclepore membranes 9 mm in diameter mounted flush on opposing walls of the channel.

The 6 micro-meter thick polycarbonate membranes have a dense distribution of 0.1 micro-meter holes and on one side is evaporated a think layer of gold which makes contact with the channel wall.

The gold layer forms one electrode of a capacitor transducer, the other being a brass electrode as shown. An ac signal of about 0.5 V peak to peak (in addition to a 100 V DC bias) results in an oscillatory motion of the membrane.

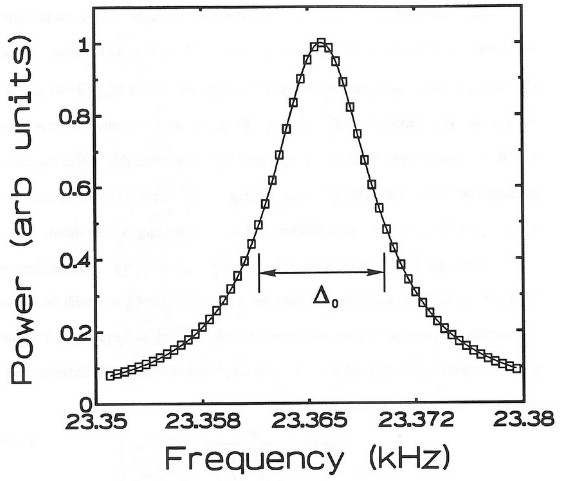
This oscillation of the membrane thus creates a variation of the relative density between normal and superfluid components. Because this density ratio is strongly temperature dependent the resulting wave is also a temperature or entropy wave and can be detected using either a similar mechanical transducer or a thermometer!



In second sound the two fluid components move in *antiphase* (above right) such that

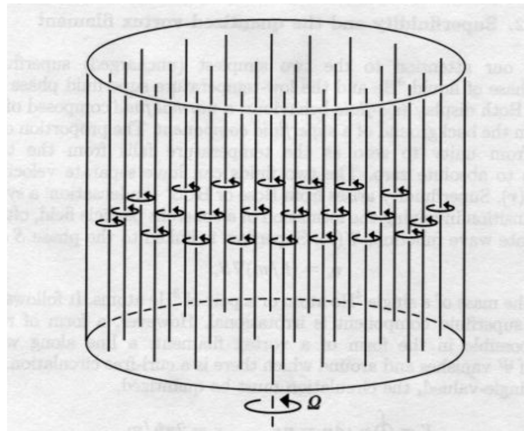
$\rho_s v_s + \rho_n v_n = 0$ and the overall density and pressure remain constant.

Second sound standing wave resonance



The channel acts as a second sound resonator. Typically a high harmonic n=50 is used to ensure plane waves, which corresponds to about 20-40 kHz. A Lorenzian resonance peak is obtained have a FWHM that is temperature dependent and typically reaches values of $\Delta_0 = 20\text{-}500$ Hz without quantized vortices in the channel.

Calibration



Vinen and Hall: in experiments with a rotating container of He II they observed an excess attentuation of second sound in direction perpendicular to rotation axis.

This extra attenuation resulted from scattering of the elementary excitations—normal fluid– by the vortex lines and was absent for second sound propagating parallel to the rotation axis.

The vorticity ω in the container was known: $\boldsymbol{\omega = 2\Omega = \kappa L}$, where Ω was the angular velocity of the container, κ the quantum of circulation, and L the length of vortex line per unit volume.

The *extra* attenuation was found to be given by:

$$\alpha_L = \frac{B\kappa L}{4u_2} = \frac{\pi\Delta_0}{u_2}\left(\frac{A_0}{A} - 1\right)$$

where B is a mutual friction coefficient, u_2 the speed of second sound, and A, A_0 are the amplitudes of the second sound resonance with and without vortices present.

We can extend this to the case of a homogeneous vortex tangle. Taking into account that vortices oriented parallel to the second sound propagation do not contribute to the excess attenuation. Then we have for the total length of quantized vortex line per unit volume:

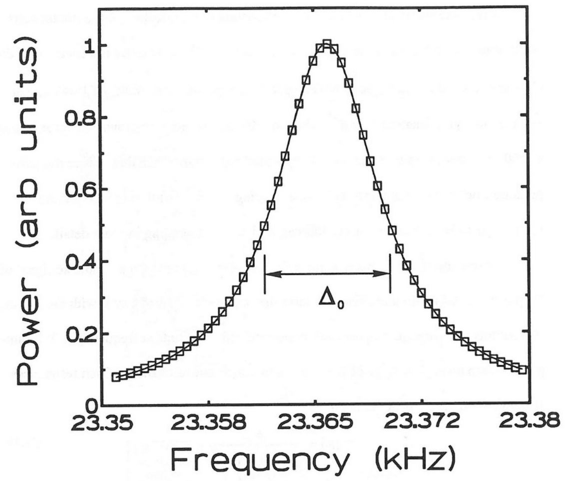
$$L = \frac{16\,\Delta_0}{B\kappa}\left(\frac{A_0}{A} - 1\right)$$

Here’s the experimental procedure:

- Park the grid at the top of the channel, establish a second sound standing wave and fit it to a Lorentzian function (make sure it’s really parked!)
- Slowly lower the grid to the bottom and wait a bit.
- Pull the grid such that the velocity profile is *linear* over most the the channel and, most of all, through the test section
- Monitor the recovery of the second sound resonance peak

The length of quatized vortex line per unit volume L is obtained from the second sound measurements through the relation

$$L = \frac{16\Delta_0}{B\kappa}\left(\frac{A_0}{A} - 1\right)$$



Where, again, A and A_0 are respectively the amplitudes of the second sound standing wave resonance peak with and without vortices present, B is the mutual friction coefficient, and Δ_0 is the FWHM (see figure at right).

Quasi-classical analysis the decay of the vortex line density

1) In classical fluid turbulence the *energy dissipation rate per unit mass* is related to the rms vorticity by the relation $\varepsilon = \nu\omega^2$. In the superfluid we assume that the energy dissipation per unit mass is given by $\varepsilon = \nu' \kappa^2 L^2$, where κ is the quantum of circulation and the coefficient ν' is an effective kinematic viscosity.

Asumming (not actually required) a Kolmogorov like energy spectrum:

$$E(k) = C\,\varepsilon^{2/3} k^{-5/3}$$

we have for the energy:

$$E = \int_{d^{-1}}^{k_d} C\varepsilon^{2/3} k^{-5/3} dk \cong \int_{d^{-1}}^{\infty} C\varepsilon^{2/3} k^{-5/3} dk = \frac{3}{2} C\varepsilon^{2/3} d^{2/3}$$

Here d is the size of the channel (the largest dimension of the measured volume)

This total energy is decreasing slowly with time :

$$\varepsilon = -\frac{dE}{dt} = -C\varepsilon^{-1/3} d^{2/3} \frac{d\varepsilon}{dt}$$

Intergrating we get

$$\varepsilon = 27C^3 d^2 t^{-3}$$

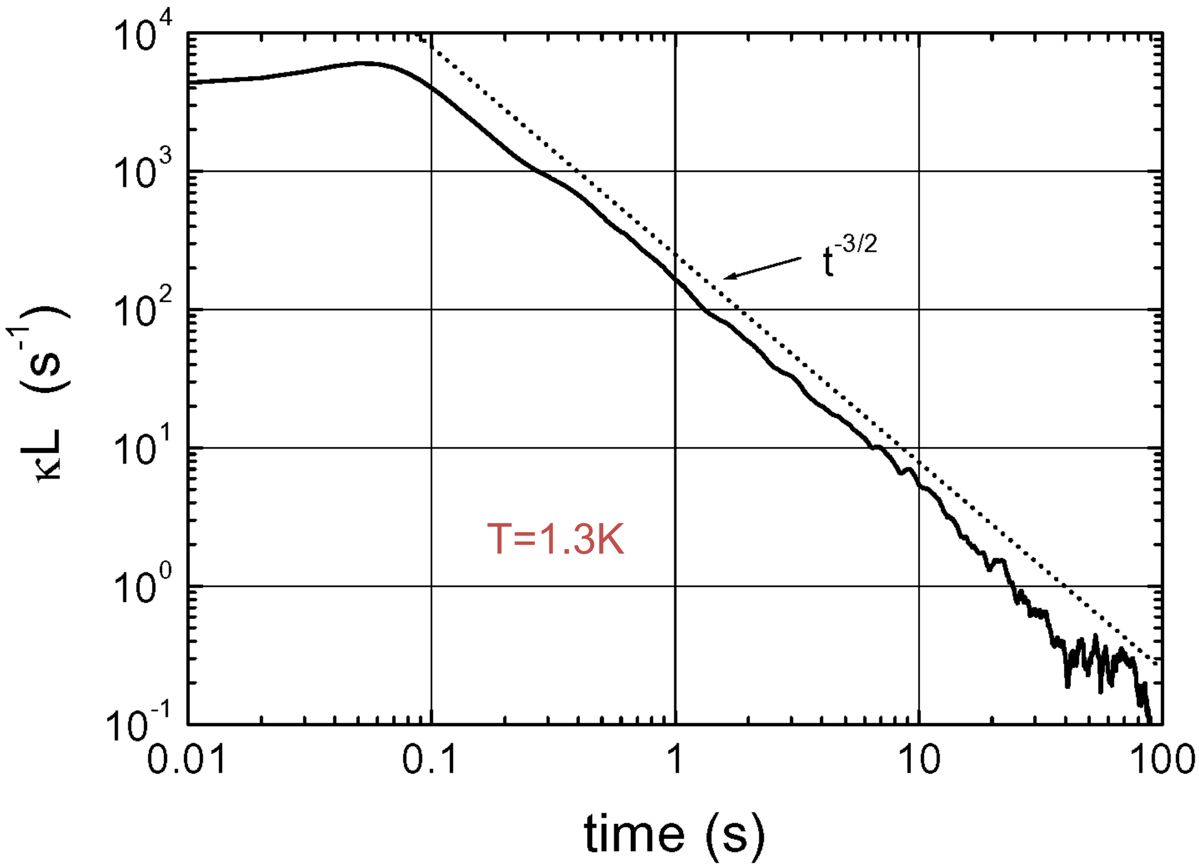
Substituting for ε:

$$\kappa L = \frac{(3C)^{3/2} d}{\nu'^{1/2}} t^{-3/2}$$

The Kolmogorov constant C can be taken equal to 1.5 which is its approximate value in classical fluid turbulence.

The only unknown quantity then is the *effective kinematic viscosity*

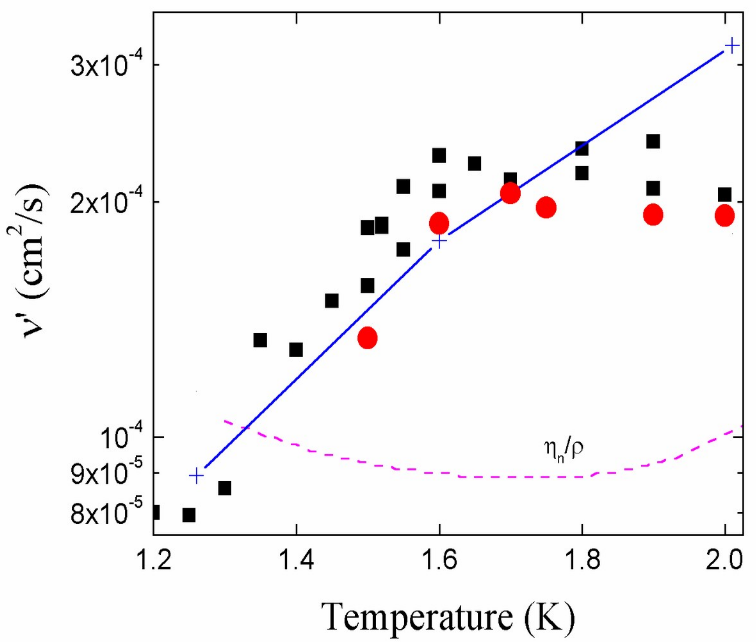
In fact, we observe precisely a -3/2 roll-off of the line density vs time



- We see that this is indeed the case, even though for the experimental data shown here at 1.3K the normal fluid fraction is nearly negligible (roughly one percent).

By fitting the decay curves to the expression for L(t) we determine the value of the only unknown: the effective kinematic viscosity ν'

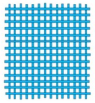
- The effective kinematic viscosity



- All data are for a mesh Reynolds number, Re_M=150k, corresponding to typical grid velocities of order 1 m s^{-1}.

- The black points are from the thesis of S.R. Stalp using a robust but rather "odd" grid:

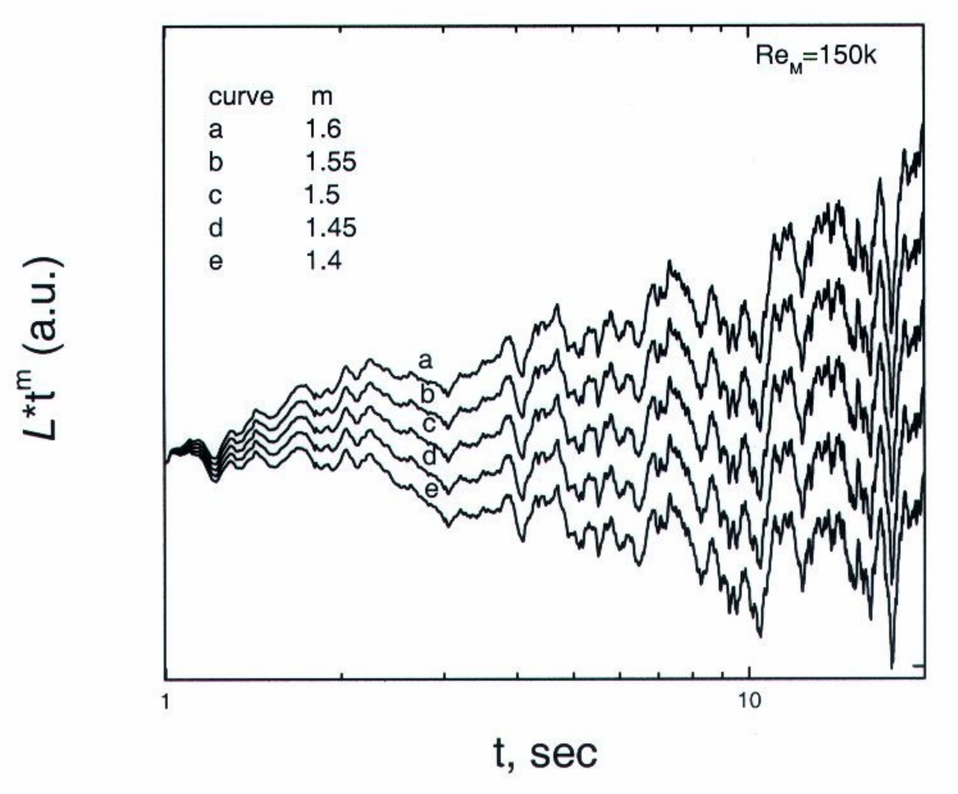
The red points (JJN, Sreenivasan and Donnelly, 2004) were taken using a more conventional, albeit delicate, grid with 13 full meshes across the channel.



The dashed line is the kinematic viscosity of the total fluid defined as the ratio of the shear viscosity of the normal component to the total density

- We note that values of the effective viscosity have the same order of magnitude as η_n/ρ, but a different temperature dependence. The order-of-magnitude agreement with η_n/ρ is probably an accident, arising from the fact that κ and η_n/ρ happen to have similar magnitudes.
  - The line connecting plusses is a theoretical result for the effective kinematic viscosity (Vinen & JJN, 2002), which is proportional to the quantum of circulation:

- Finally, how good was the assumption that there was a -3/2 power law rather than something else?



- Clearly, curve “c”, corresponding to the power 3/2, best represents horizontality in this normalized plot.

- Note: we can derive the expression for the decaying line density without explicitly invoking Kolmogorov. We take the expression for the energy dissipation rate that we considered before (lecture 2):

$$\varepsilon = C_\varepsilon \frac{u^3}{\ell}$$

- where the constant $C_\varepsilon \simeq 0.5$

- We assume that the length scale grows with time just as in classical turbulence becoming comparable to the channel width d. We then can write:

$$u^2 = \left[\frac{\varepsilon d}{C_\varepsilon}\right]^{\frac{2}{3}}$$

- Taking $\varepsilon = -\frac{\mathrm{dE}}{\mathrm{dt}} = -\frac{d}{\mathrm{dt}}\left(\frac{3}{2}u^2\right)$
- we have $-\varepsilon\left(\frac{d}{C_\varepsilon}\right)^{2/3} = \varepsilon^{-1/3}\frac{d\varepsilon}{\mathrm{dt}}$

- Integrating and using $\varepsilon = \nu'$
- we obtain

$$\kappa L = \frac{27^{1/2} d}{\nu'^{1/2} C_\varepsilon} t^{-3/2}$$

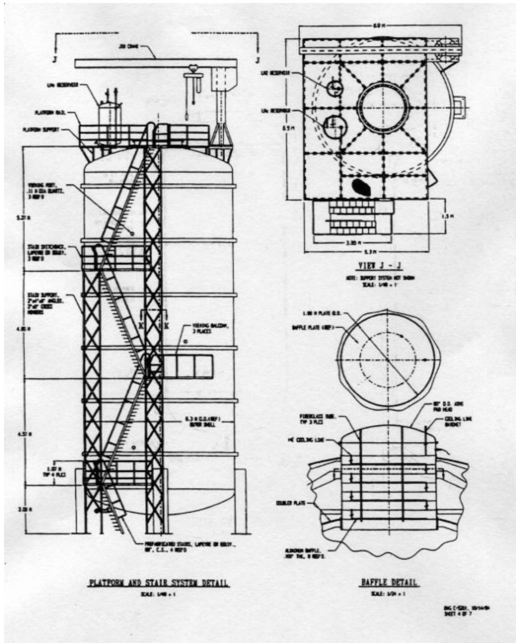
- equivalent to what we found before with $C_\varepsilon \simeq 0.5$

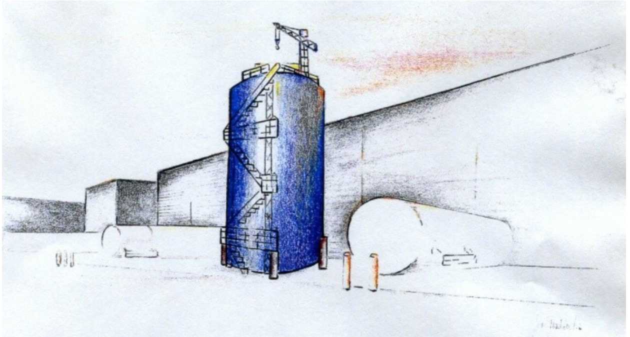
A proposed 10m high convection cell capable of Ra~10^{21} nearly comparable to that characteristic of solar convection.

Inside cell dimensions
D = 5m, L = 10m,
Max volume ~ 25,000 gallons of liquid helium equivalent

Outside dimensions
~7 m dia and ~20 m high

Refrigeration needed
< 200 W





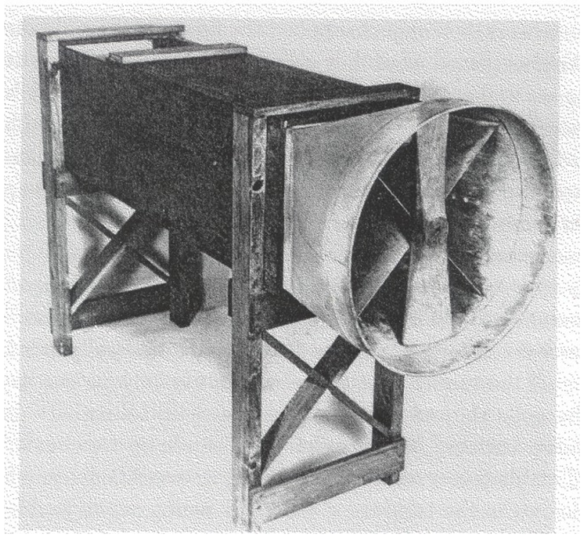
RHIC, BNL

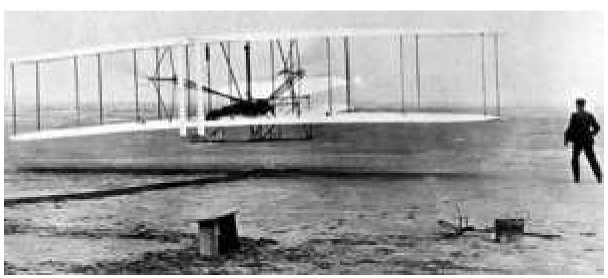
Huge accelerator facilities like CERN or BNL would have plenty of liquid helium on hand, used to cool superconducting magnets.

Testing applications

An historical digression

The Wright Brothers: 1st successful application of wind tunnel data



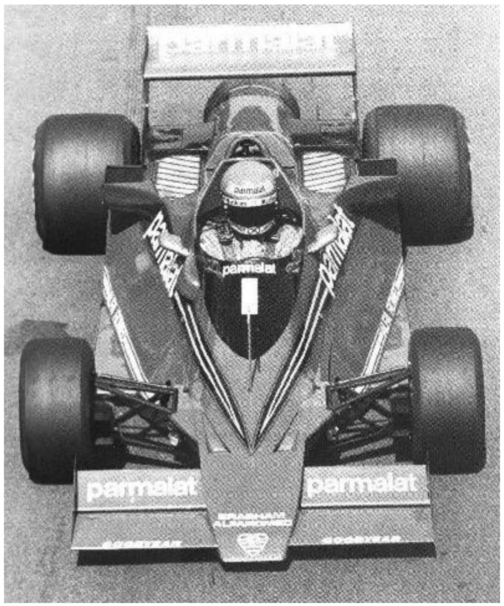


"We directed the air current from an old fan in the back shop room into the opening of the wooden box. Occasionally I had to yell at my brother to keep him from moving even just a little in the room because it would disturb the air flow and destroy the accuracy of the test."

"Over a two month period we tested more than two hundred models of different types of wings. All of the models were three to nine inches long. We finally stopped our wind tunnel experiments just before Christmas, 1901. We really concluded them rather reluctantly because we had a bicycle business to run and a lot of work to do for that as well."

---Wilber Wright

On each little aircraft wing design we tested we located the center of pressure and made measurements for lift…" Wilber Wright



The main "driving" force behind innovations in testing is for applications where you hope that "lift" never gets you off the ground…

At Old Dominion University resides the largest University-operated wind tunnel in the world. "Customers include NASA, the Navy, racecar teams (principally NASCAR)…"

Cryogenic testing facilities

In aerodynamic testing, one measures lift, drag and moments on a model and infers the corresponding values on the prototype. Since *Re* can often be very large in practical situations (of the order 10^8 or 10^9 for commercial aircraft or modern submarines), it is difficult to match the prototype Reynolds number if the same fluid is used for model testing.

Helium has an advantage over highly compressed air because the dynamic pressure $(1/2)\rho U^2$ is substantially smaller for a given Reynolds number, and hence the helium flow can be expected to exert significantly less force on the models.

Another possibility with helium is the use of powerful superconducting magnetic balance and suspension systems both to orient models without the external arm or “stinger”, and to measure forces on them.

- A cryogenic tunnel

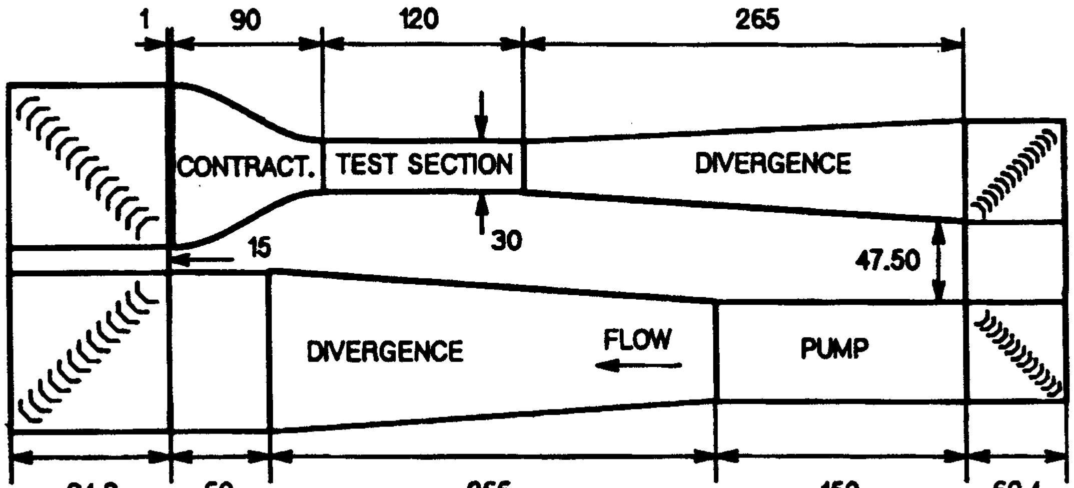
- The National Transonic Facility (NTF) at the NASA Langley Research Center, operating since 1984. Cryogenic liquid nitrogen is sprayed and evaporated into a gas that is accelerated through the tunnel's test section up to a Mach number of 1.2. The 150-m long tunnel is powered by a 100 MW turbine motor. The figure on the right shows the giant vanes that help air flow around a corner.

$$Re \equiv UL\rho/\eta$$

Helium wind tunnel

- A 30 cm helium tunnel could be considered “table-top” compared to the large wind tunnels of NASA. A 125 cm model would reach comparable Re to any of them.

Liquid helium tunnel



NASA AMES