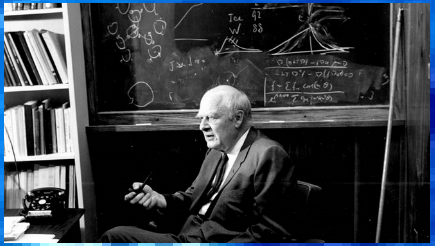
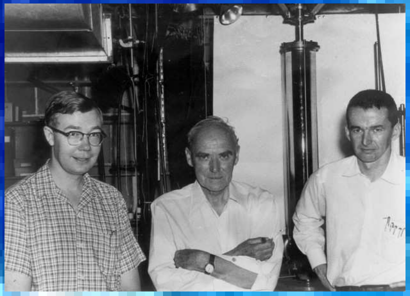
Turbulence near absolute zero

A slightly historical perspective



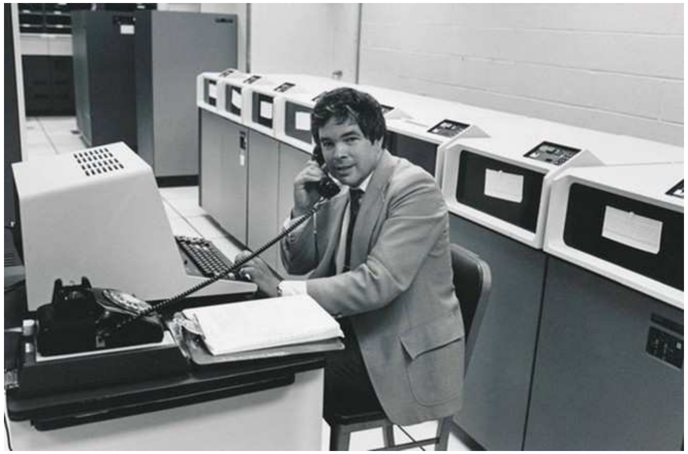


J. Niemela
ICTP, Trieste

5 centuries later….

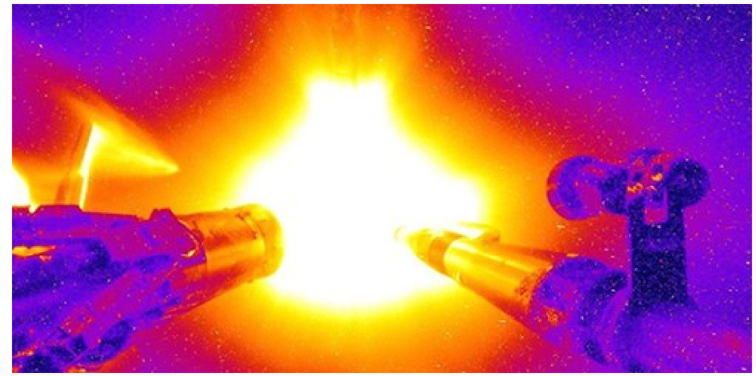
“Finally, there is a physical problem that is common to many fields, that is very old, and that has not been solved. It is not the problem of finding new fundamental particles, but something left over from a long time ago – over a hundred years. Nobody in physics has really been able to analyze it mathematically satisfactorily in spite of its importance to the sister sciences.
It is the analysis of circulating or turbulent fluids.”
--Richard Feynman, Nobel Laureate Physics

And from Ken Wilson in his Nobel lecture: “….the entire problem of ***fully developed turbulence****….[has] defeated analytic techniques up till now.”*

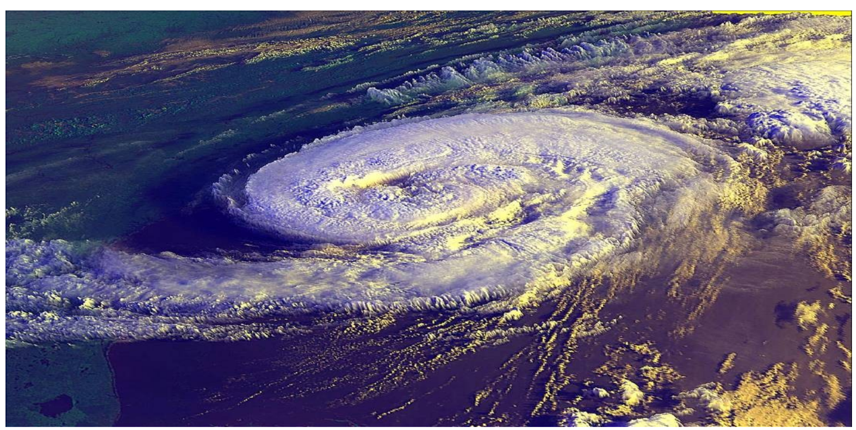


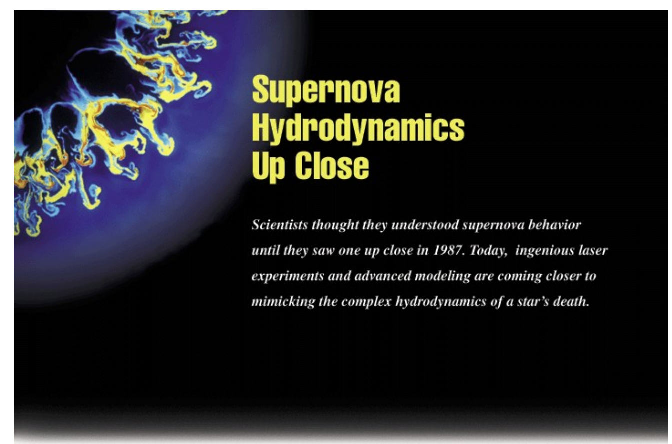
Turbulence is a paradigm problem for complex systems and plays an important role in a wide range of contexts

In engineering design, heat transfer, inertial confinement fusion



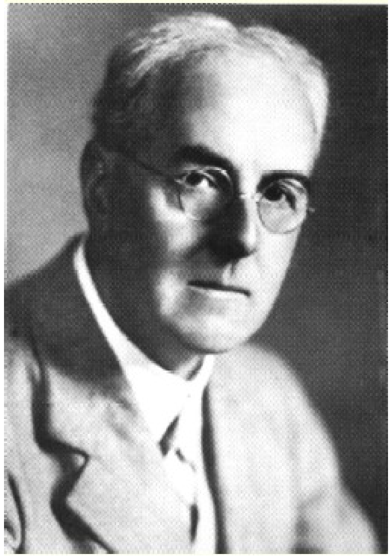
Also: pollutant dispersion in the earth's atmosphere and oceans, large scale geophysical and astrophysical phenomena





An interpretation in line with the Richardson cascade:

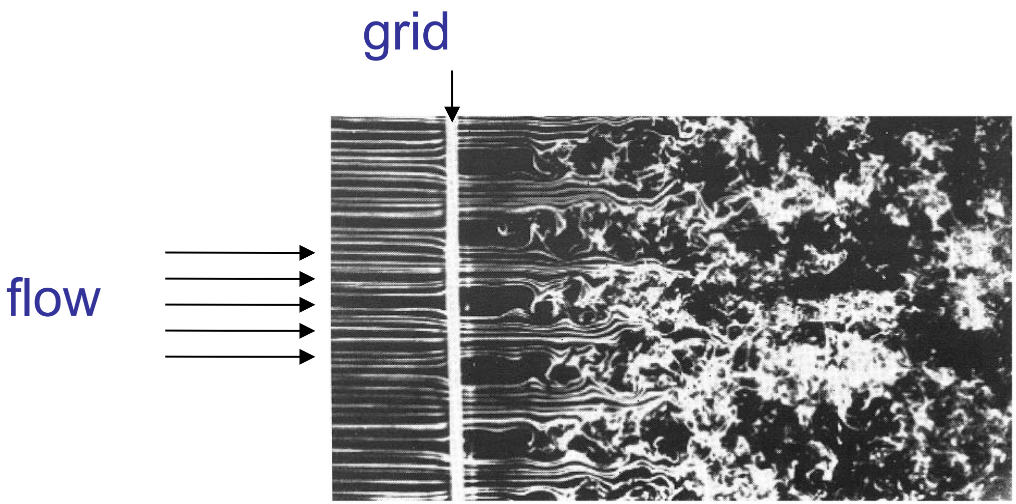
"Big whirls have little whirls that feed on the velocity
And little whirls have lesser whirls and so on to viscosity"

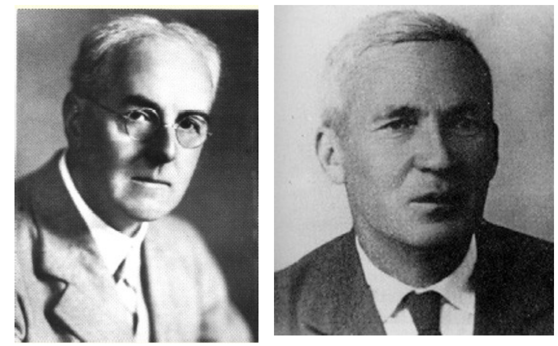


Richardson

Leonardo da Vinci: "...the water has eddying motions, one part of which is due to the principal current, the other to the random and reverse motion…the small eddies are almost numberless..."

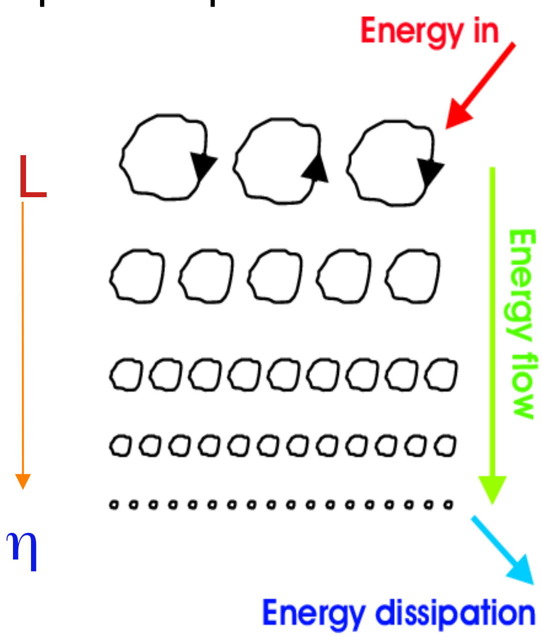
(Approximately) homogeneous isotropic turbulence





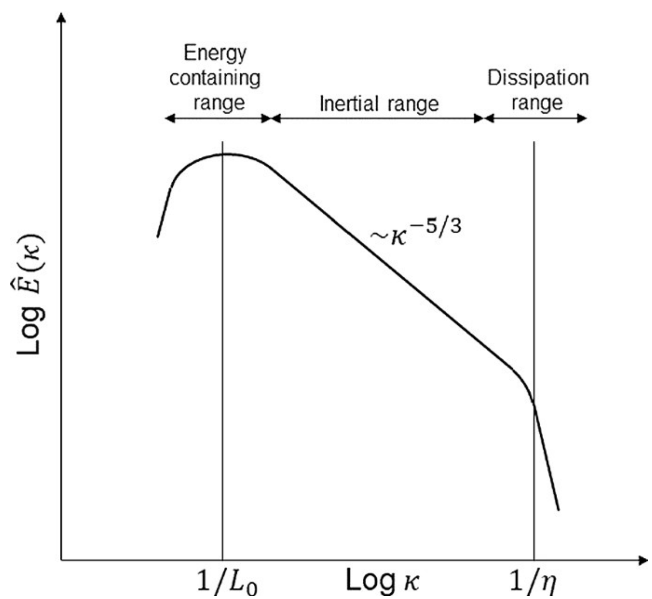
Richardson Kolmogorov

Spatial representation



$$\frac{L}{\eta} = \mathrm{Re}_L^{3/4}$$

Wavenumber representation

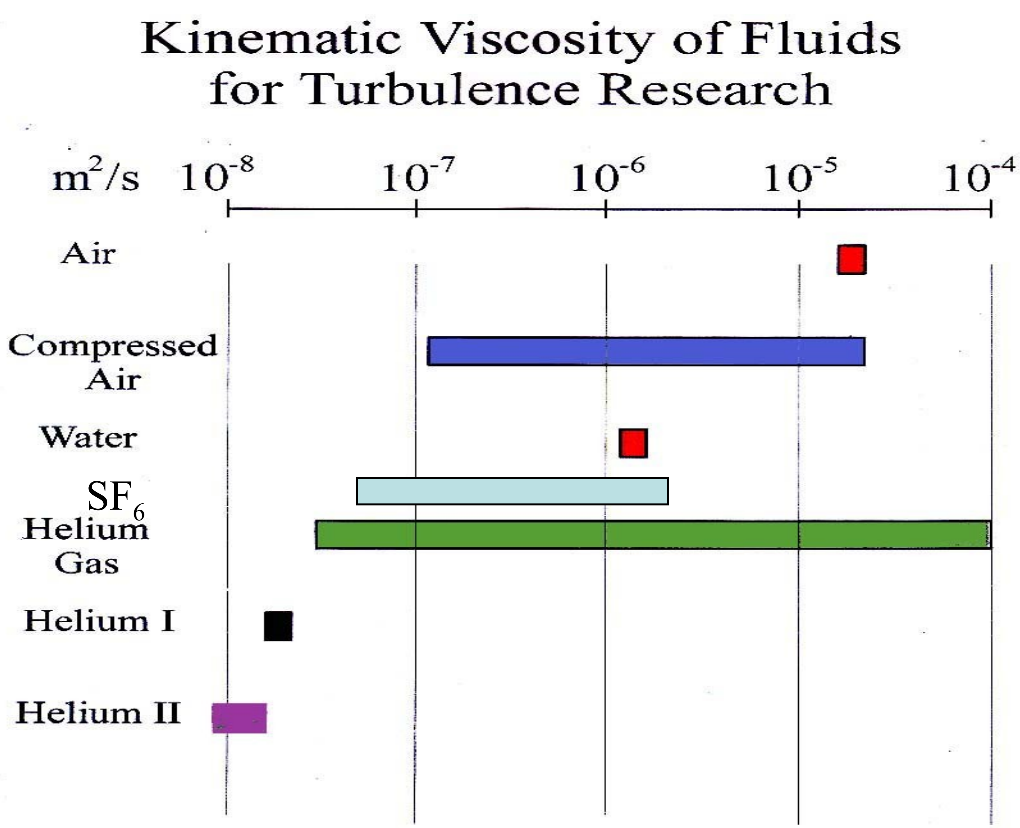


It is reasonable to look for *universal properties* of turbulence as Re→∞

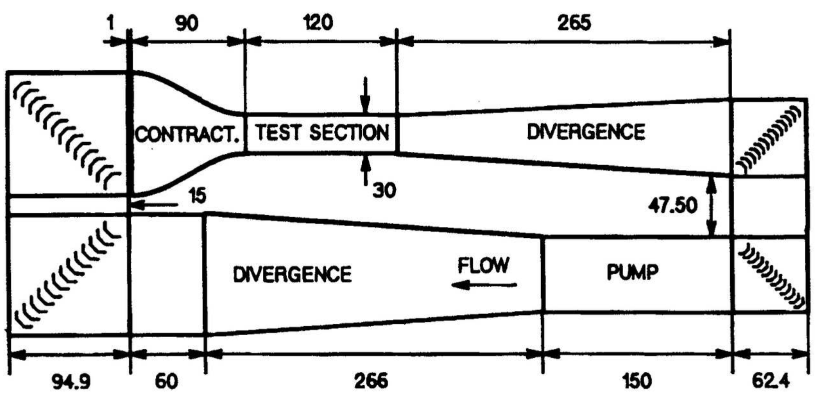
Note: Re→∞ is not the same as Re=∞

Approximating Re→∞ experimentally

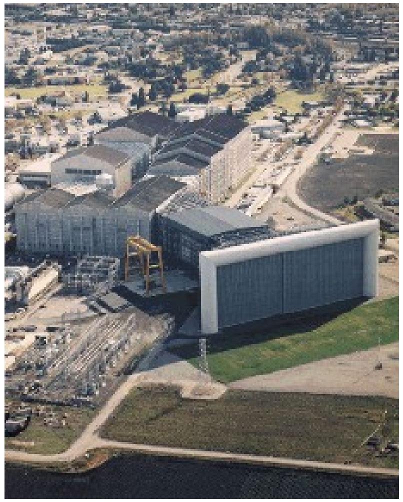
$$Re = \frac{UL}{\nu}$$



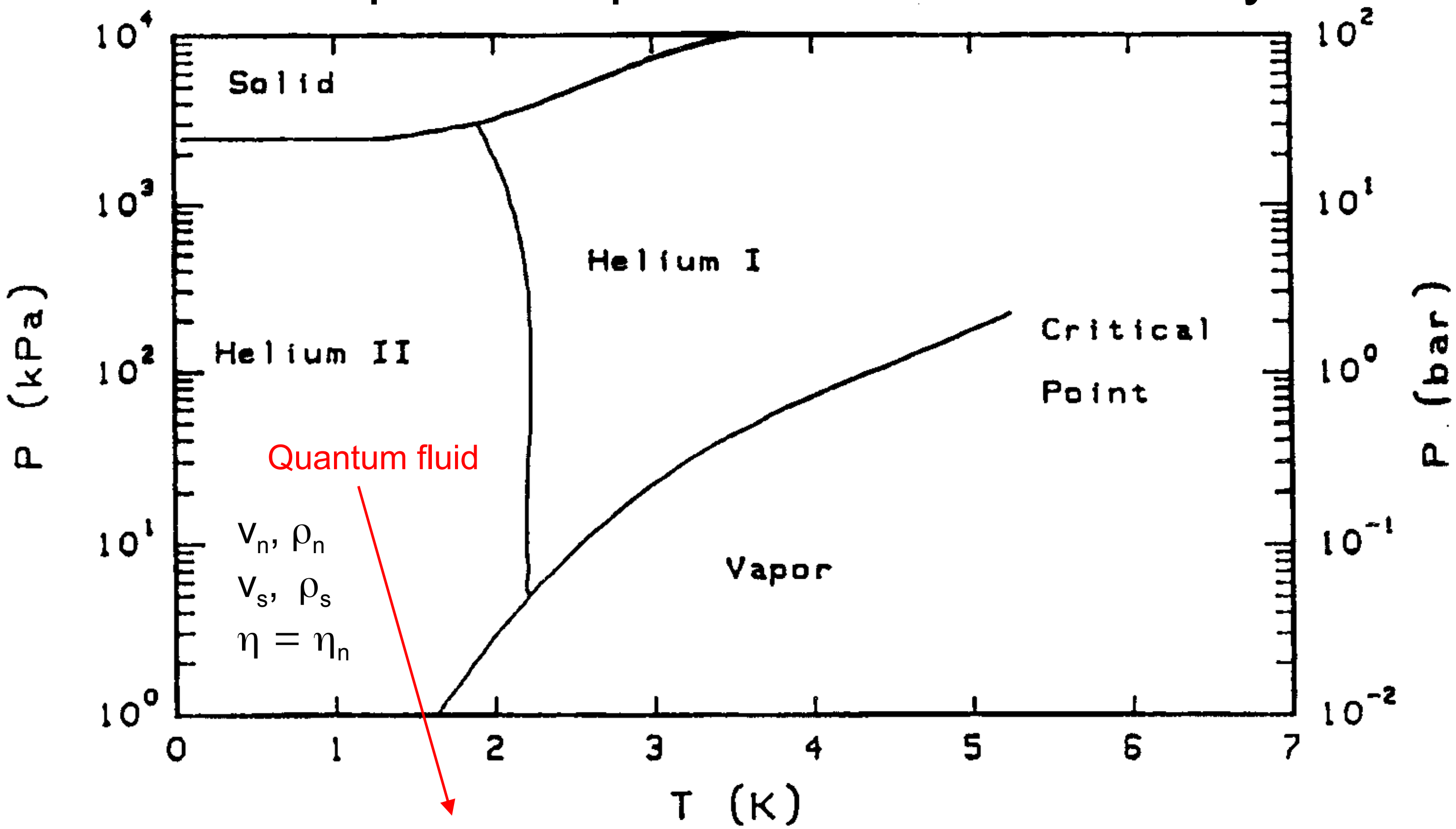
A room-sized superfluid helium wind tunnel



Comparable to NASA Ames tunnel:



The quantum phase of helium...briefly



Thermal DeBroglie wavelength is larger than mean inter-atomic distance

Macroscopic wave function ψ governed by single particle wave equation with phase **S**(r)

$$\psi(\mathrm{r}) = \psi_0 \mathbf{e}^{\mathrm{i}\mathrm{S}(\mathrm{r})} \qquad \mathbf{v}_s = \frac{\hbar}{\mathrm{m}_4} \nabla \mathbf{S} \longrightarrow \nabla \times \mathbf{v}_s = 0$$

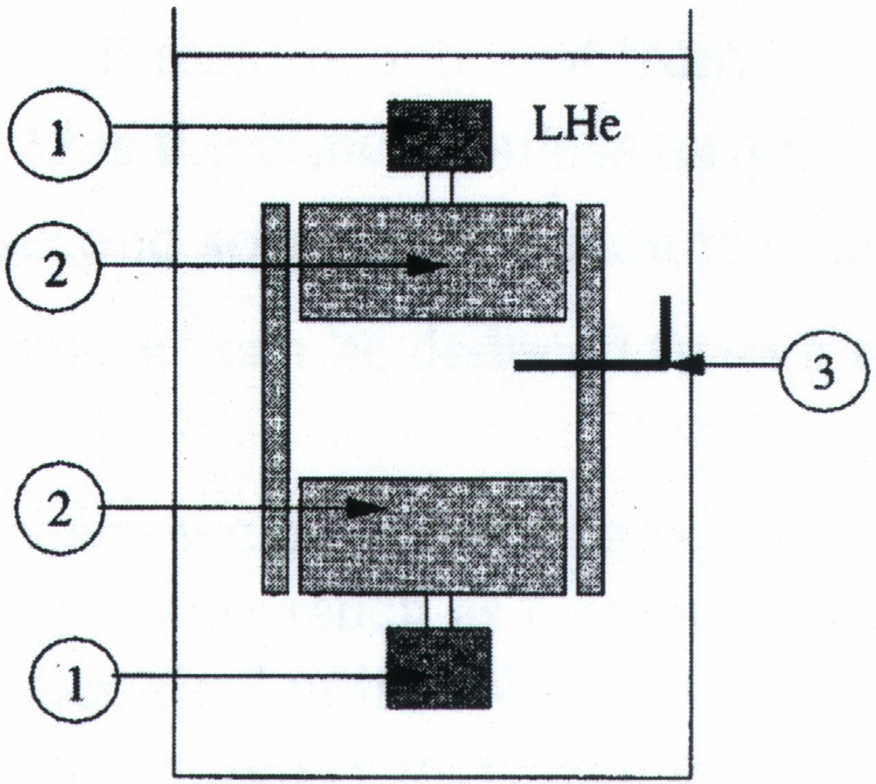
Superfluid has finite circulation about a line *singularity* (hollow vortex core):

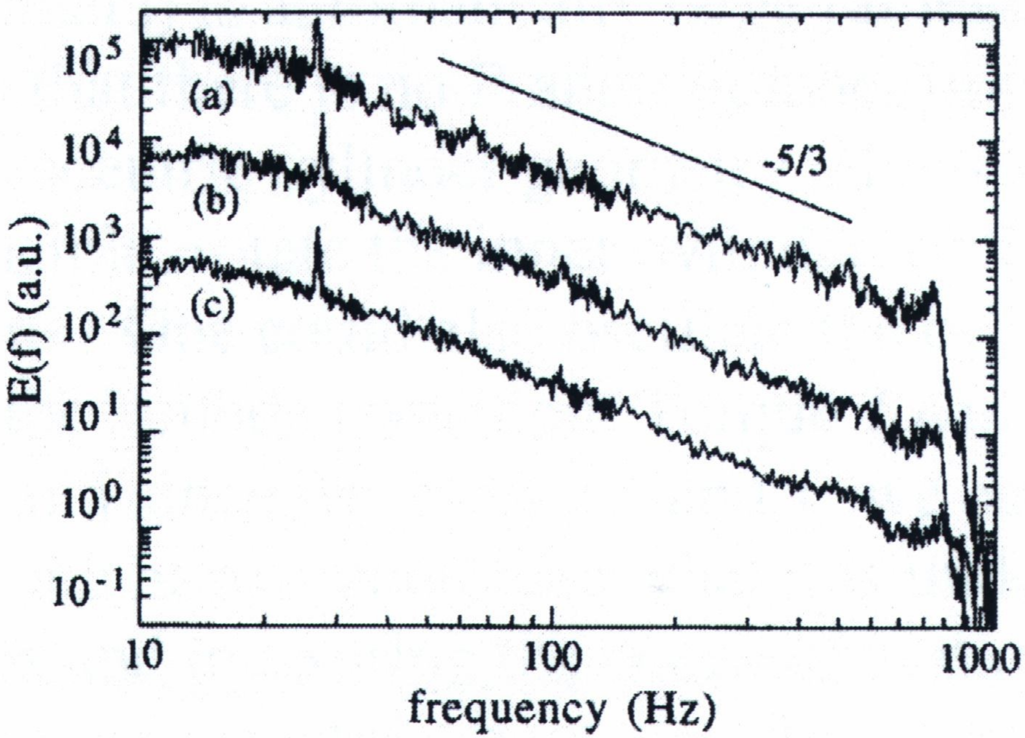
$$\longrightarrow \quad \kappa = \oint v_s \cdot \mathrm{dl} = \frac{\hbar}{m_4} \Delta \mathrm{S} = \mathrm{n} \frac{h}{m_4}$$

Feynman: Turbulence can exist as a (polarized) tangle of singly-quantized vortex filaments

Superfluid “washing machine” (Maurer and Tabeling 1998)

Evidence of classical energy cascade





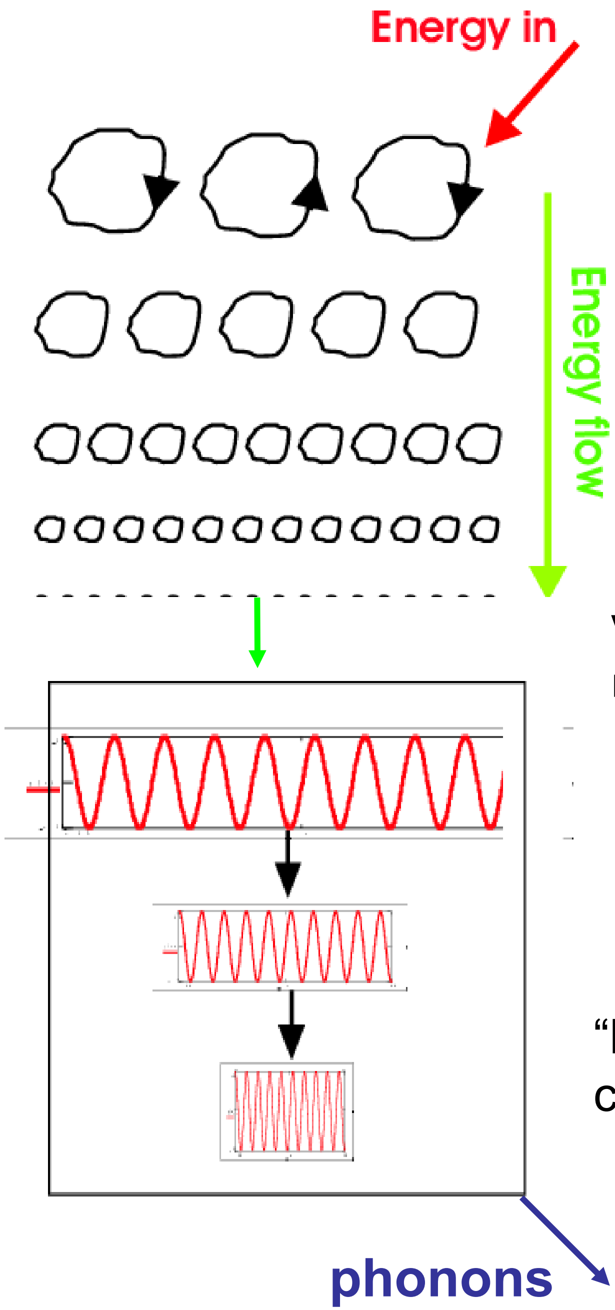
Pressure measurements

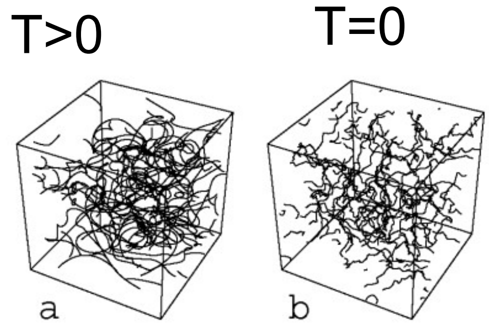
a) Helium I

b) Helium II $\rho_n \sim \rho_s$

c) Helium II $\rho_s/\rho \sim 1$

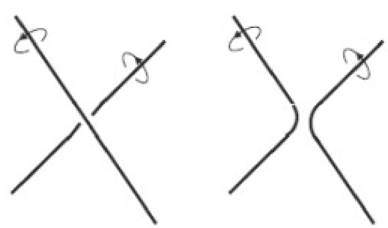
What about scales smaller than the inter-vortex line spacing?





Classical Richardson cascade on scales greater than vortex line spacing.

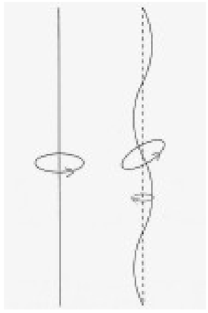
Vortex reconnections are needed to bridge the gap



energy flow

Kelvin wave cascade on scales less than inter-vortex line spacing .

"Plucked" vortex cores emit sound



energy flow

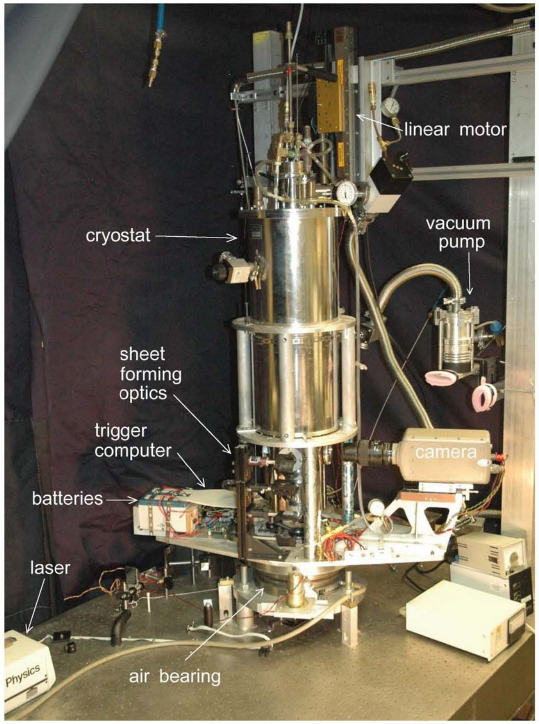
Phonons

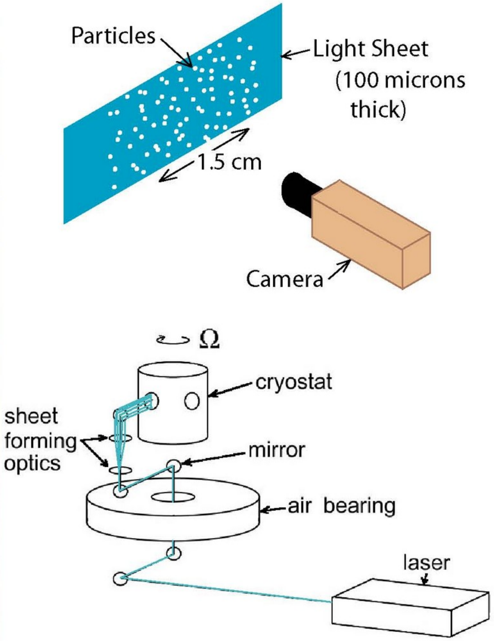
Visualizing quantized vortices and their reconnections

Dissertation: Bewley
Bewley, et al (*Nature* 2006)
Paoletti, et al (*PRL* 2008)

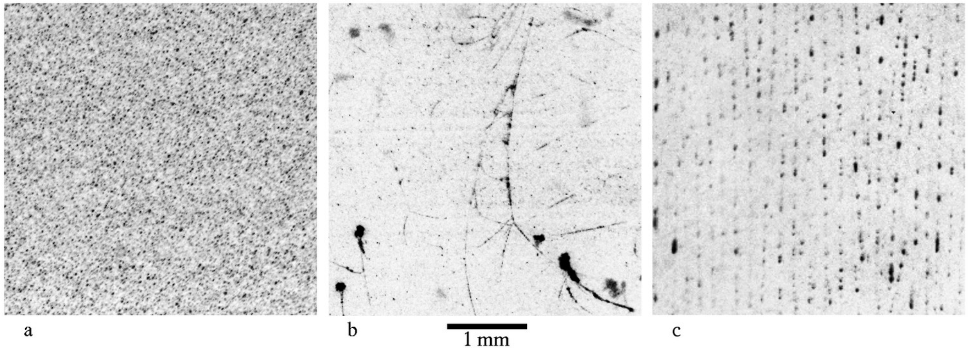
JJN: "Reconnecting to superfluid turbulence"
http://physics.aps.org/articles/v1/26

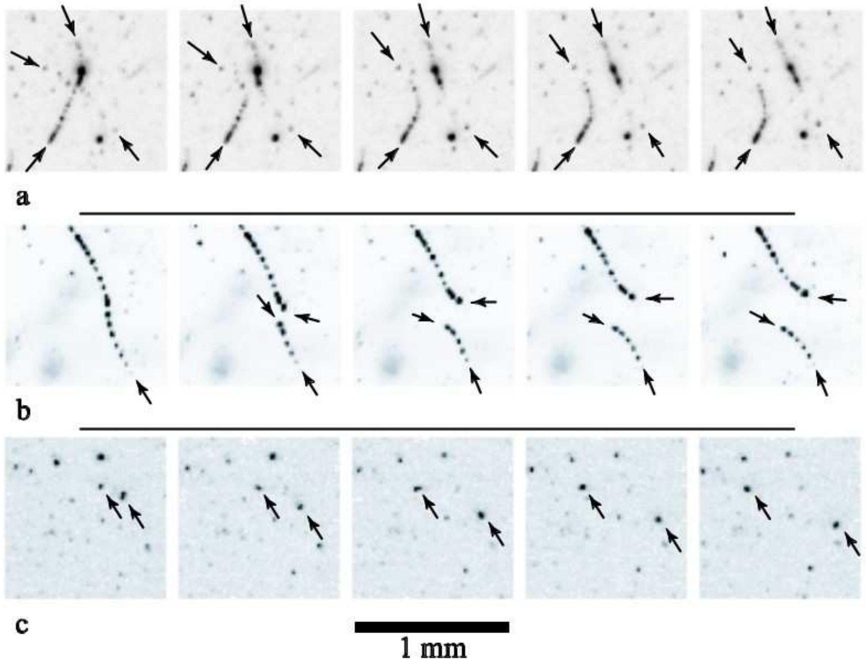
Apparatus



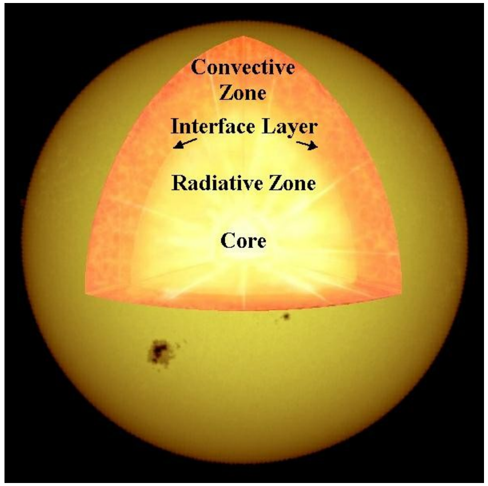


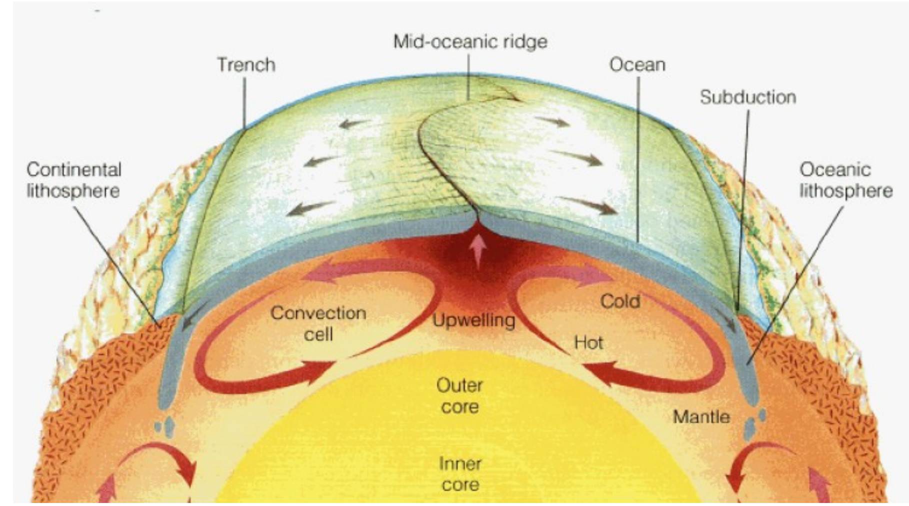
Laser Beam routing



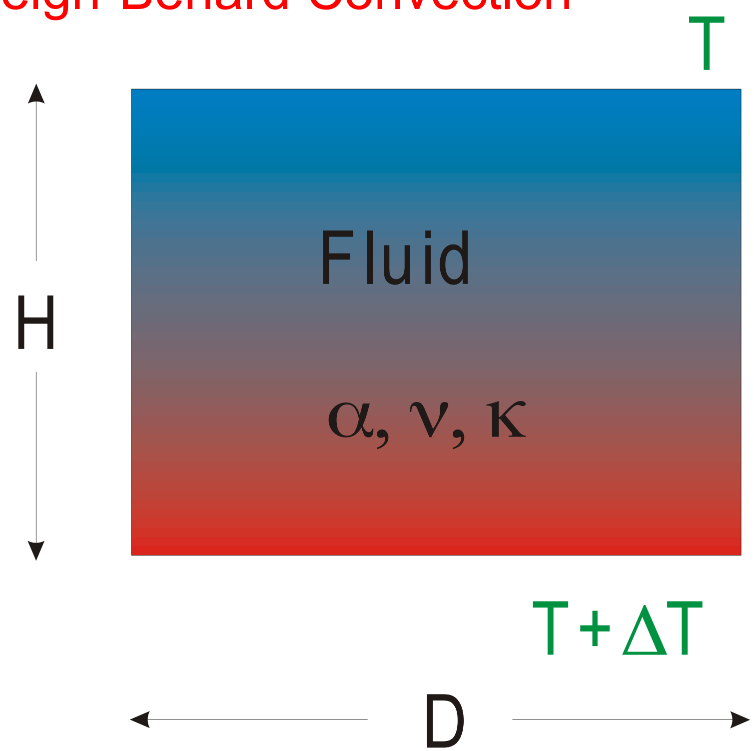


Most large-scale turbulent flows in nature are buoyancy-driven





Rayleigh-Benard Convection



Control parameters

$$Ra = \frac{g\alpha\Delta T H^3}{\nu\kappa}$$ Rayleigh number

$$Pr = \frac{\nu}{\kappa}$$ Prandtl number

$$\Gamma = \frac{D}{H}$$ Aspect ratio

The normalized heat transport: Nusselt number

$$Nu = Q\left(\frac{k\Delta T}{H}\right)^{-1}$$

Q = applied heat flux; *k* = fluid thermal conductivity

Nu = *f* (Ra; Pr; Γ; …)

At "asymptotically" high Ra:

$$Nu = 0.073Ra^{1/3}$$

Willem Malkus, Ed Speigel

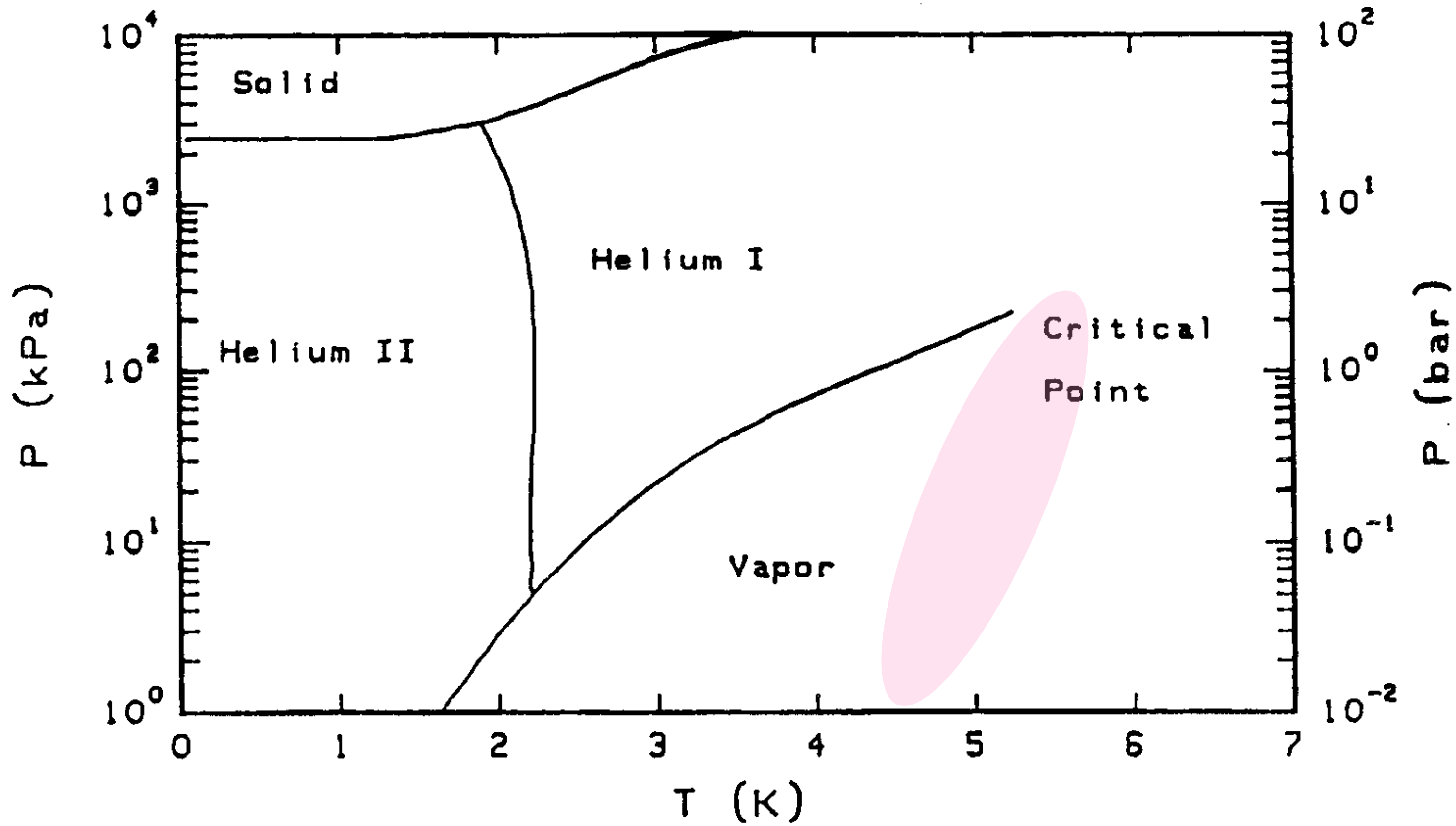
$$Nu = CPr^{-1/4}\left[Ra/log(Ra)^3\right]^{1/2}$$

Bob Kraichnan

$$Nu = D\left[Ra^{3/2}ln(Ra)^{3/2}\right]^{1/5}$$

Jack Herring (*with input from Busse, Howard, Roberts, Stewartson, Malkus, etc…)*

Pushing Ra, Re to the limit



$$\mathrm{Ra}=g \cdot \left(\frac{\alpha}{\nu\kappa}\right) \cdot \Delta T \cdot H^3$$

cgs units

4.4 K , 2 mbar:
$\alpha/\nu\kappa = 5.8 \times 10^{-3}$

5.25 K, 2.4 bar:
$\alpha/\nu\kappa = 6.5 \times 10^{9}$

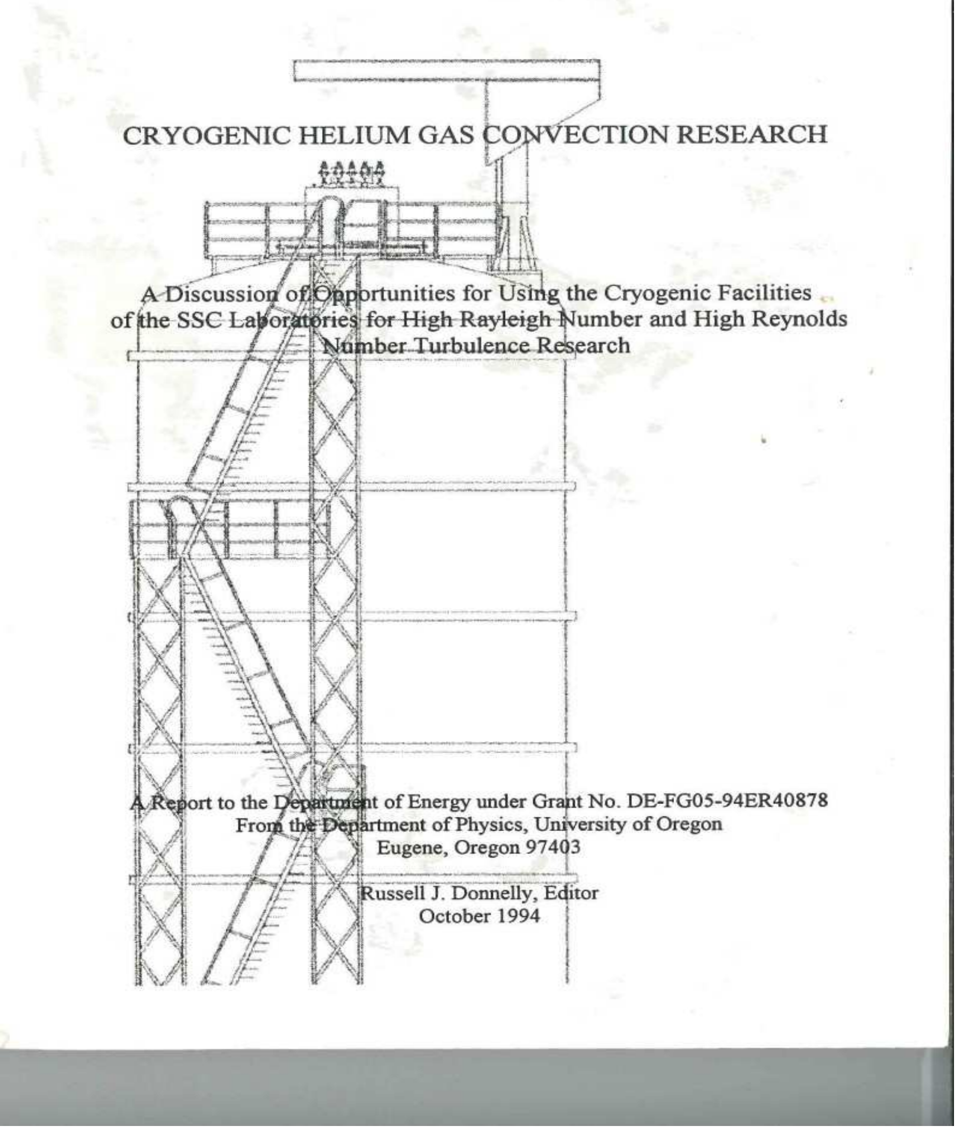
Water: $\alpha/\nu\kappa \sim 10$

Air: $\alpha/\nu\kappa \sim 0.1$

12 orders of magnitude in a control parameter up to the highest values ever attained

A BIG idea:
Ra = 10^{21}

See: Annual Reviews of Condensed Matter Physics 2021 for the whole story



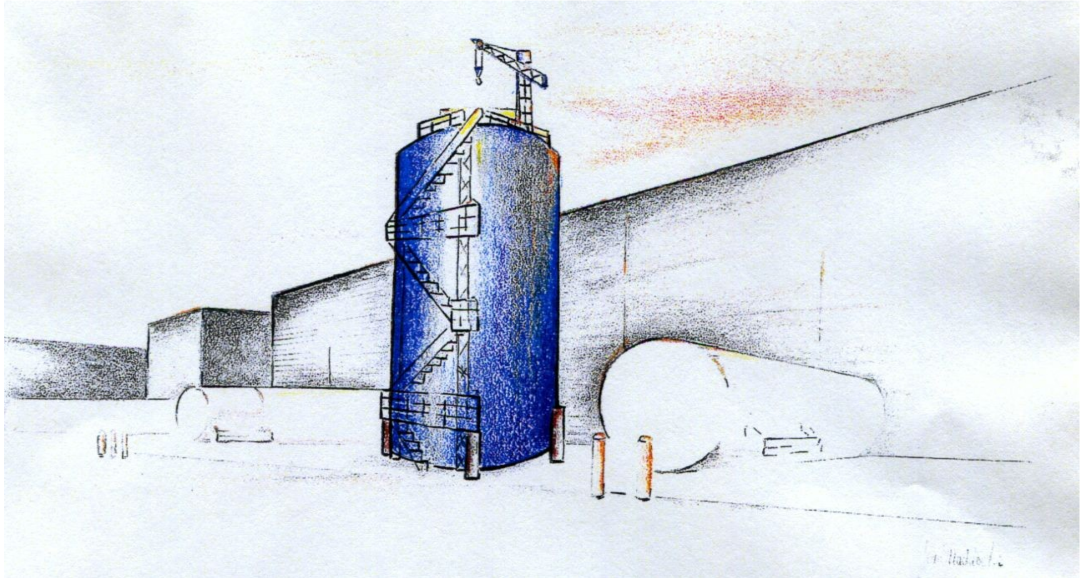
CRYOGENIC HELIUM GAS CONVECTION RESEARCH

A Discussion of Opportunities for Using the Cryogenic Facilities of the SSC Laboratories for High Rayleigh Number and High Reynolds Number Turbulence Research

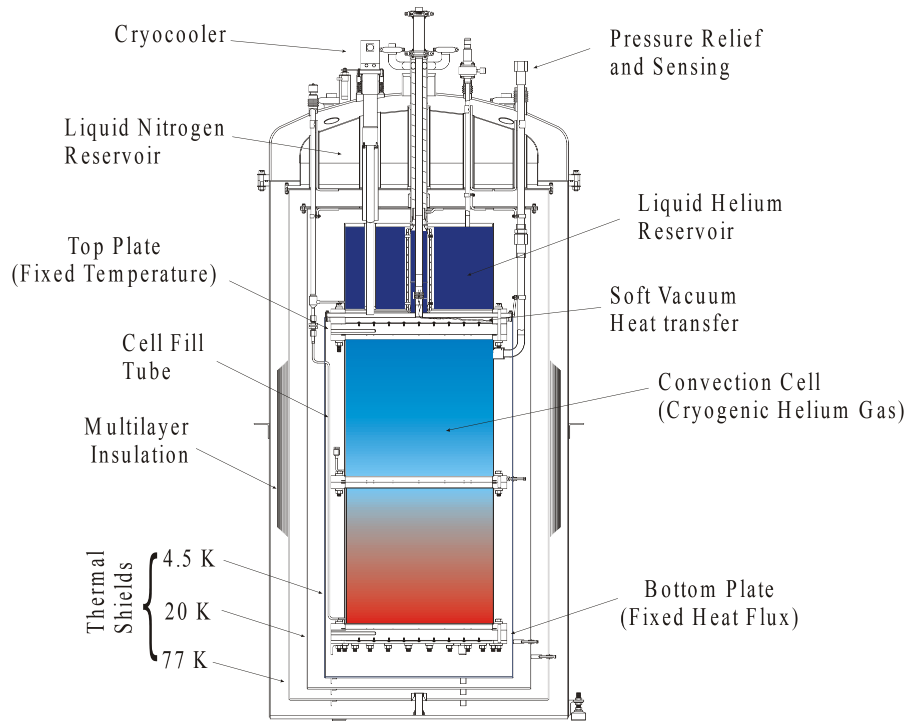
A Report to the Department of Energy under Grant No. DE-FG05-94ER40878
From the Department of Physics, University of Oregon
Eugene, Oregon 97403

Russell J. Donnelly, Editor
October 1994

Artist's conception of exp. at the SSC site in Texas

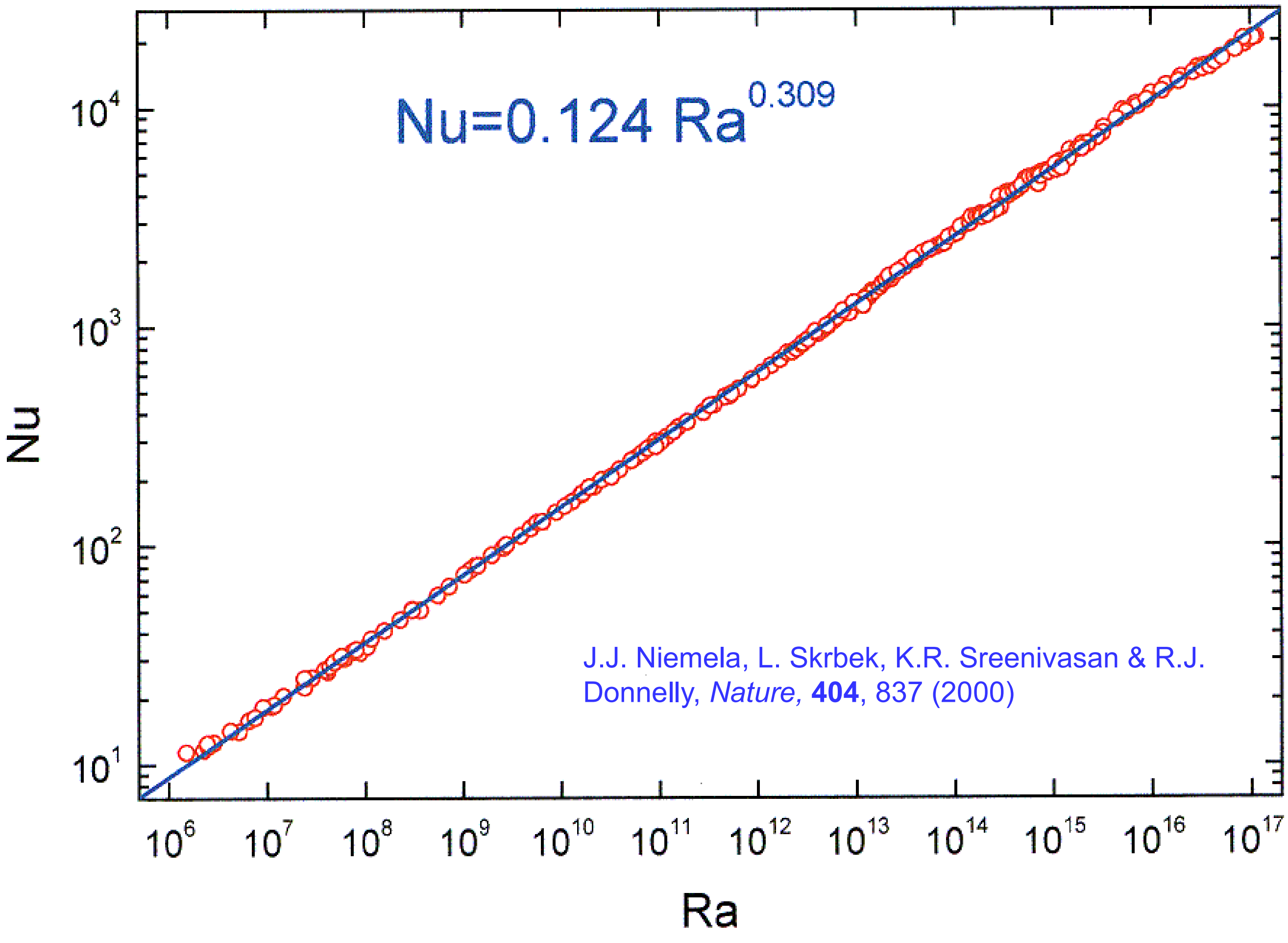


A working prototype- at ICTP

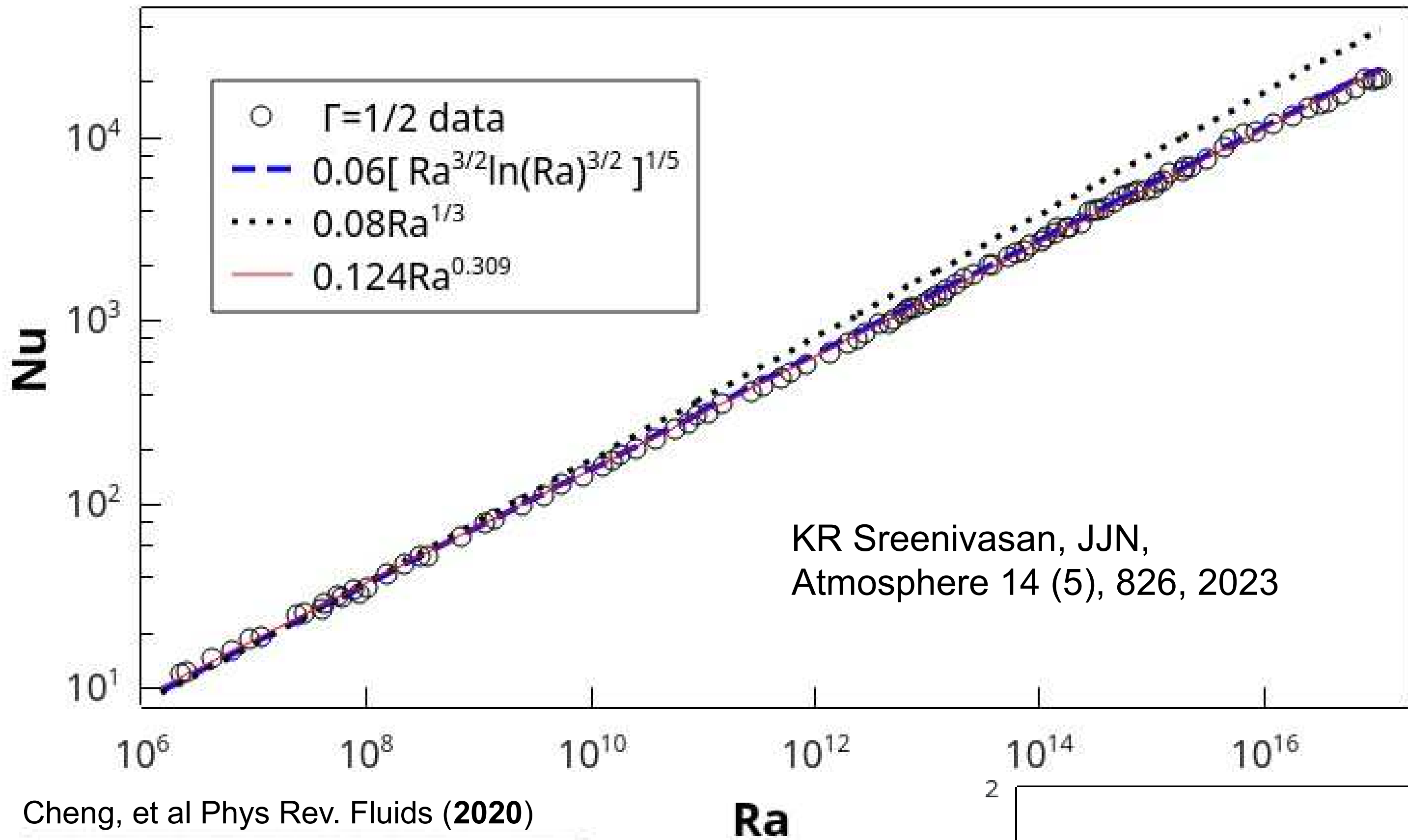


Scaling of the Nusselt Number up to Ra ~ 10^{17} (Re~10^{8})

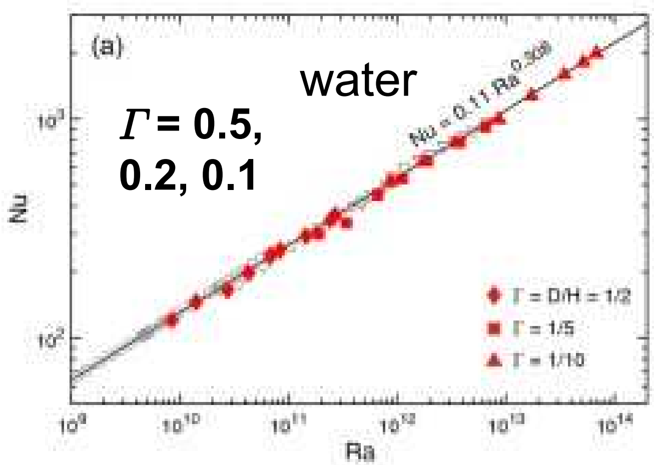
Conductivity enhancement by a factor of 20,000!



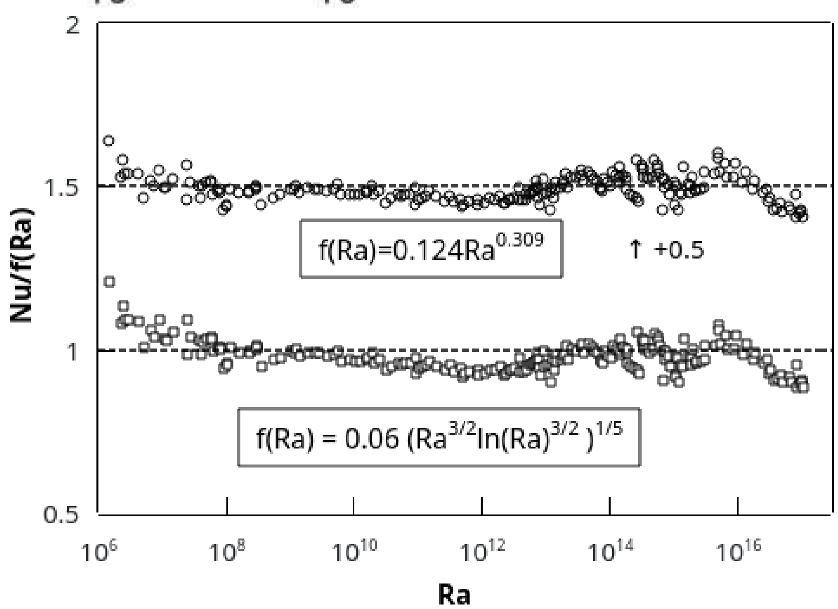
Excellent agreement *surprisingly* with the **weakly nonlinear** theory of Jack Herring as well as recent water experiments in a 4m cell



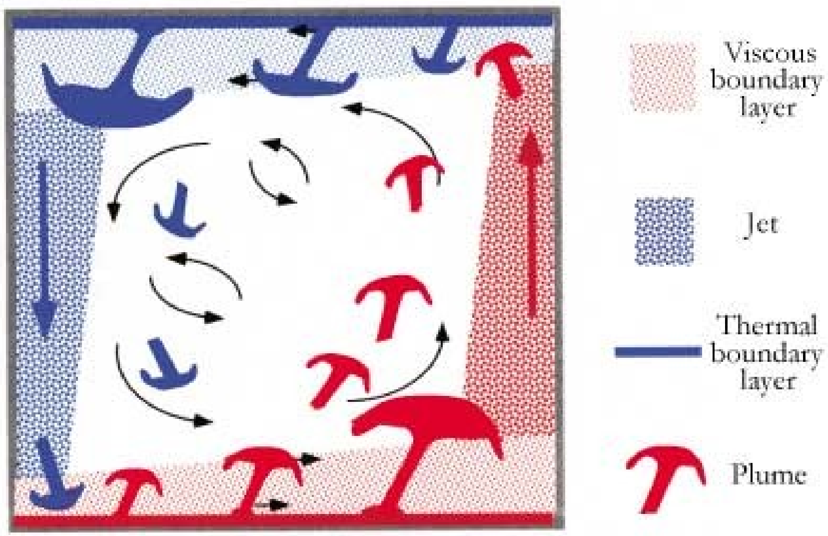
Cheng, et al Phys Rev. Fluids (**2020**)



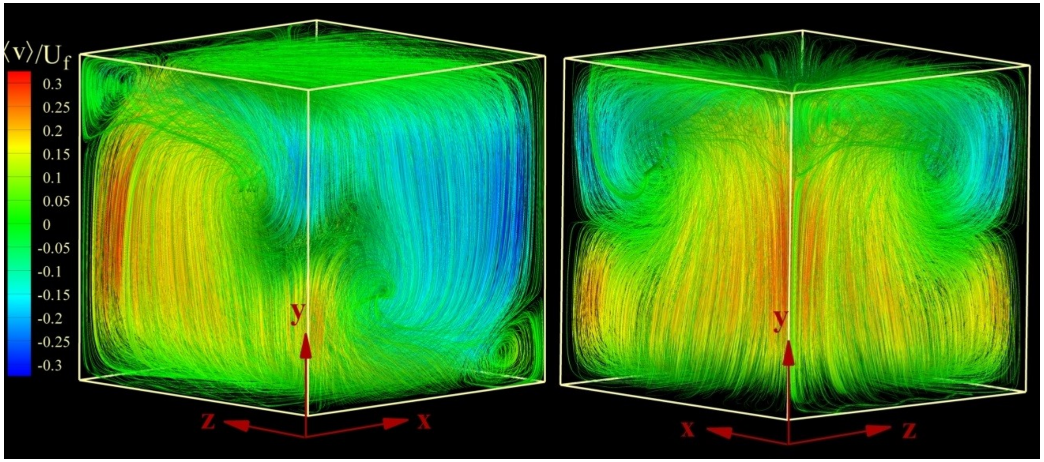
$Nu = 0.11(+-0.02)\ Ra^{0.308(+-0.005)}$



What turbulent convection looks like and why Herring's model works:



After L. Kadanoff, Physics Today 2001



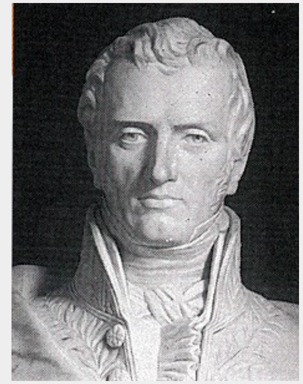
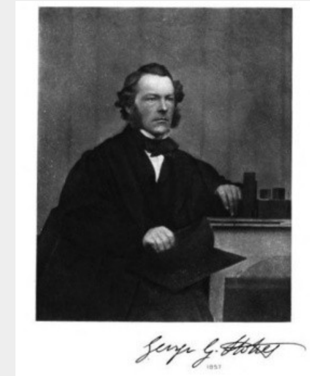
LES simulations of N. Foroozani in a cube

Herring: "*The physical picture of free boundary convective process predicted by the model is that of a large-scale motion dominating the central region between the conducting plates. This large-scale motion sweeps with it the temperature fluctuation field whose main variances occur in a thin boundary layer of vertical extent 1/Nu. The horizontal scale of both the dominant motion and the temperature fluctuation field is comparable to the distance between the conducting plates*."

This actually describes precisely the experimental observations with rigid boundaries as shown also in the simulations of N. Foroozani and is indeed the situation we might expect in a weakly nonlinear state, at least for *confined* systems. It would have been advantageous to have a larger system if only to have less confinement at such large values of Ra.

THANKS!

Navier-Stokes equations

(Navier-Stokes)

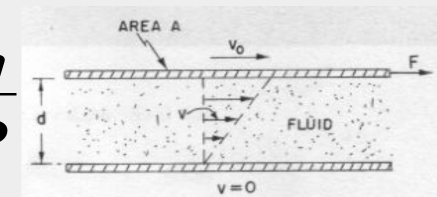
Reynolds

Non-dimensional momentum equation

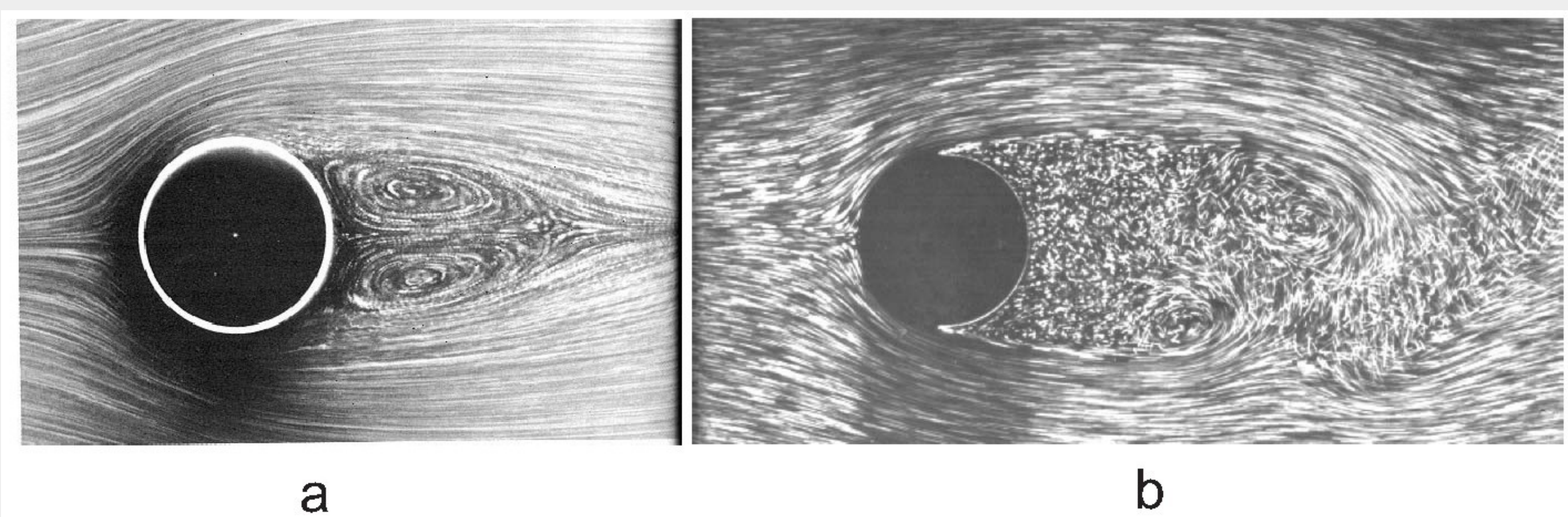
$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\,\mathbf{v} = -\nabla p + \frac{1}{Re}\nabla^{2}\mathbf{v}$$

$Re = UL/\nu$

$$\nu = \frac{\eta}{\rho}$$



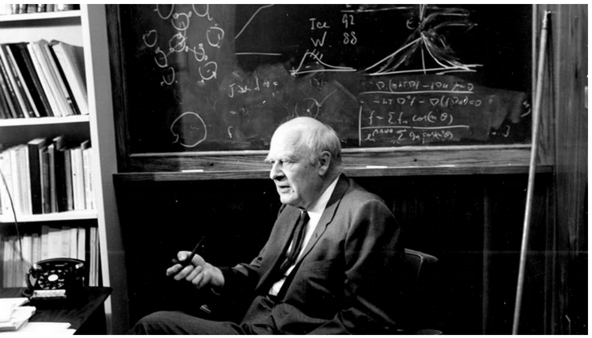
Turbulence occurs when Re is **large**



Flow past a circular cylinder. (a) Re = 26. (b) Re = 2000.

In the early days of low temperature fluid dynamics:

Onsager, Feynman, Chandrasekhar (including their young colleague Russ Donnelly) became interested in the *hydrodynamics* of superfluid helium4, including the nature of **turbulent flows.**



"Vortices in a suprafluid are presumably quantized; the quantum of circulation is h/m, where m is the mass of a single molecule."-- L. Onsager (1949)

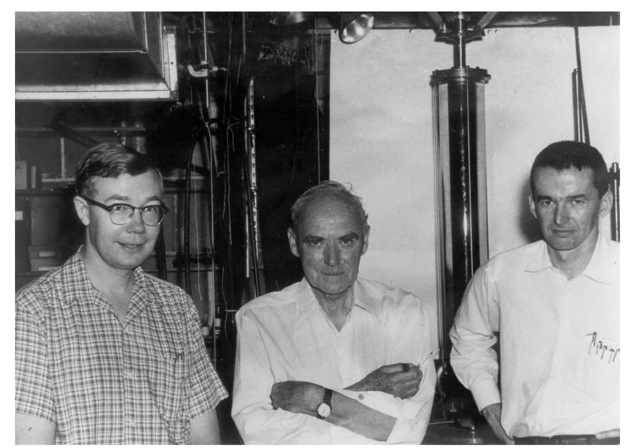
"In this way I was led to the idea that when liquid He flows, the vorticity is quantized and all the vorticity is along certain lines of one unit each. And when I was all finished I found a reference to a remark made by Onsager in a meeting in Italy, to a lecture on a different subject, in which he mentions the same solution!" -- R. Feynman (1957)

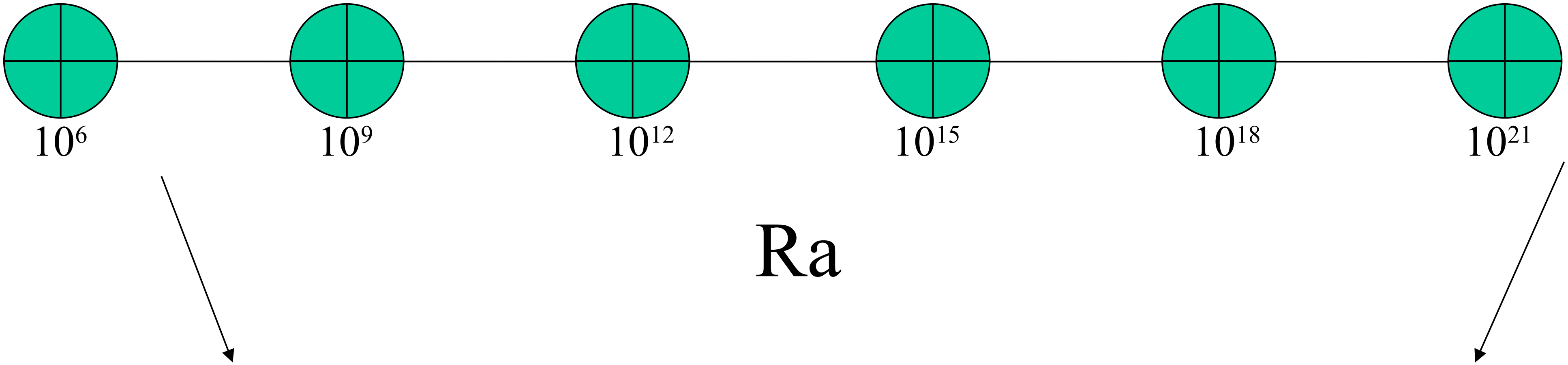
The hydrodynamic stability of helium II between rotating cylinders. I

By S. Chandrasekhar, F.R.S.
Enrico Fermi Institute for Nuclear Studies, University of Chicago

and R. J. Donnelly
Institute for the Study of Metals, University of Chicago

(*Received 6 March 1957*)




10^6
10^9
10^{12}
10^{15}
10^{18}
10^{21}
Ra