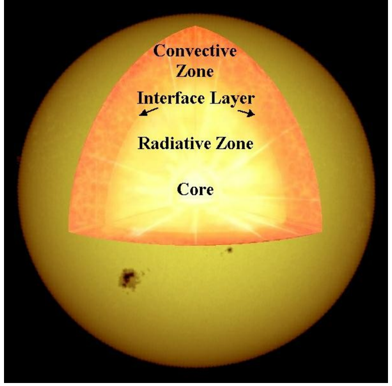
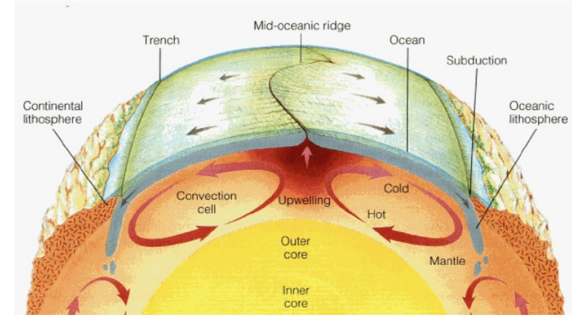
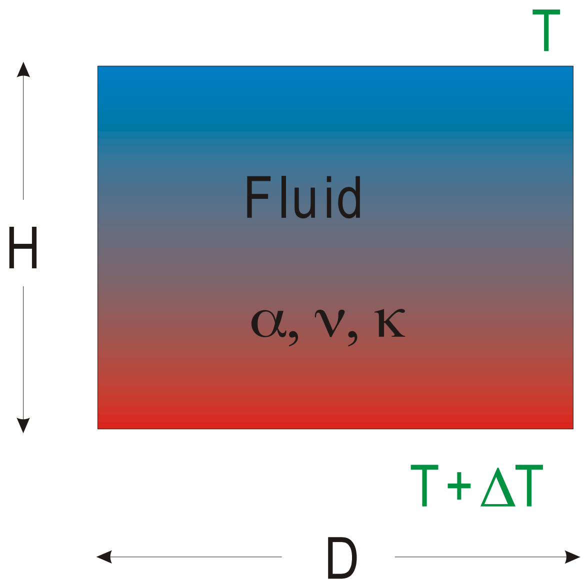


Buoyancy-driven flows common in Nature

We *cannot* hope to replicate in a laboratory every aspect of such systems



Rayleigh-Benard convection



α = (isobaric) coefficient of thermal expansion

$\nu =$ kinematic viscosity

$\kappa =$ thermal diffusivity

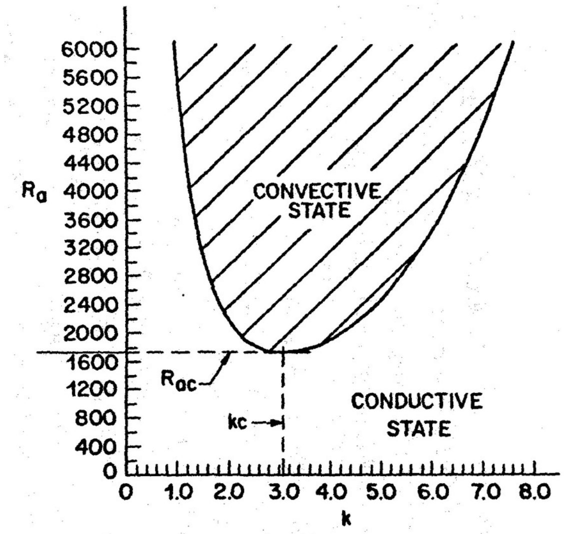
Time scales: $\tau_\vartheta = H^2/\kappa$, $\tau_\nu = H^2/\nu$, $\tau_B = H/V_{FF} = \left(\dfrac{H}{\alpha g \Delta T}\right)^{1/2}$

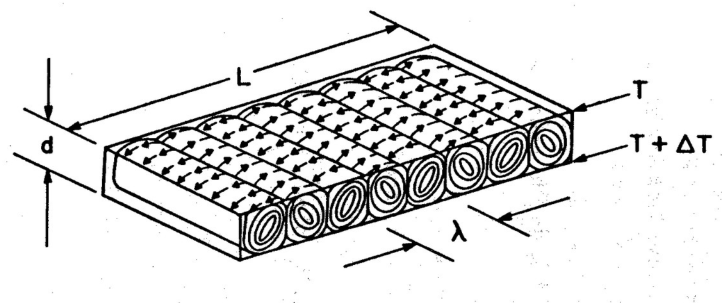
Control parameters for convection

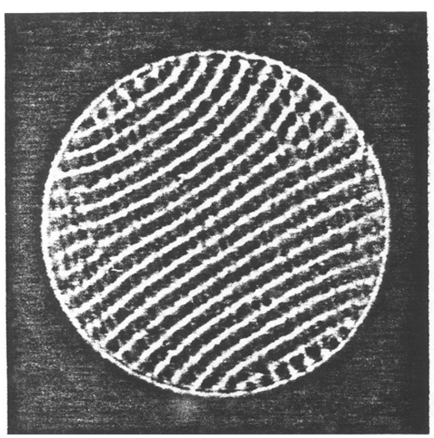
$$Ra = \frac{\alpha g H^3 \Delta T}{\nu\kappa} = \frac{\tau_\vartheta \tau_\nu}{\tau_B{}^2} \qquad Pr = \frac{\nu}{\kappa} = \frac{\tau_\vartheta}{\tau_\nu} \qquad \Gamma = \frac{D}{H}$$

RBC near onset: For the most part, a completely understood problem*

*subject to some discussion





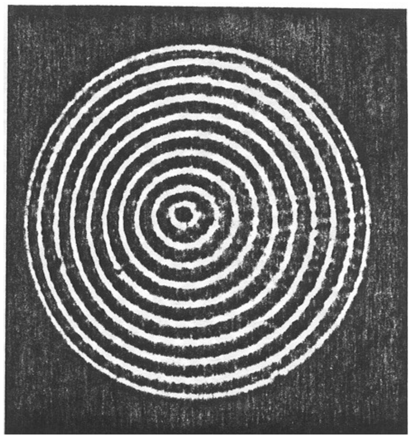


We even understand the diference between

this

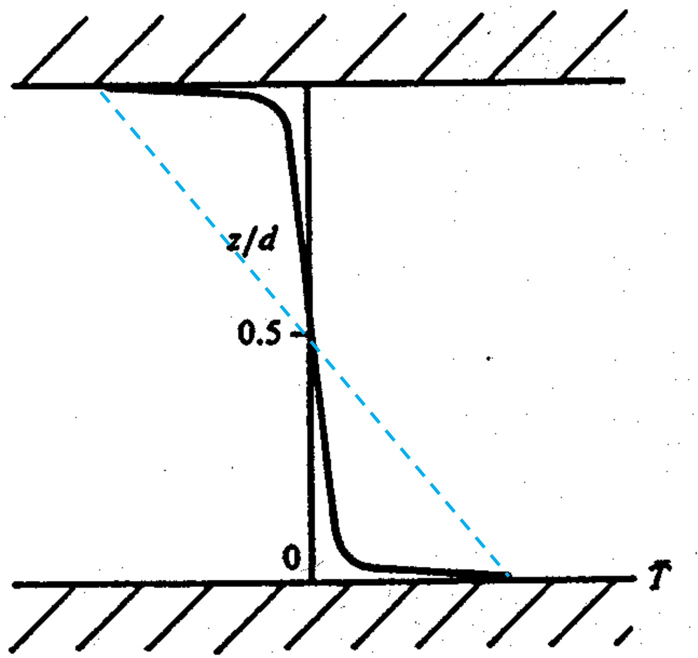
and

this



RBC at very high Ra:

Thermal boundary layers at the upper and lower walls are highly stressed regions giving rise to "plumes."



The temperature gradient is all at the wall! At high Ra in experiments the boundary layer is typically of order 100 micrometers.

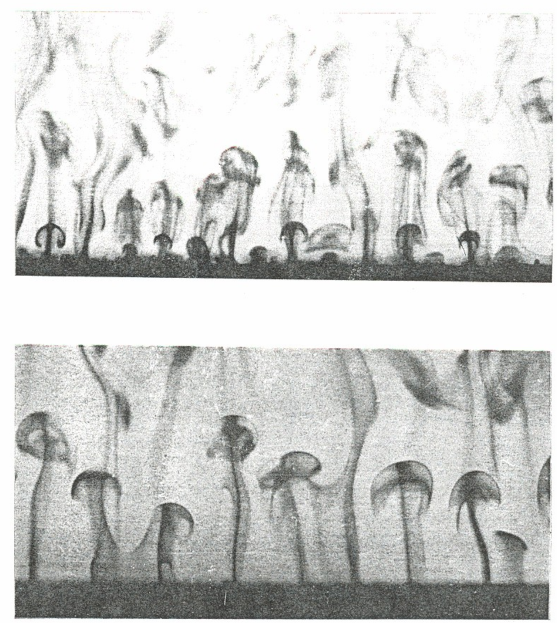


FIGURE 1. Photographs of thermals rising from a heated horizontal surface.

Plumes in water

Sparrow, Husar & Goldstein J. *Fluid Mech*. **41**, 793 (1970)

The normalized heat transport: Nusselt number

$$Nu = Q\left(\frac{k\Delta T}{H}\right)^{-1}$$

Q = applied heat flux; *k* = fluid thermal conductivity

Nu = *f* (Ra; Pr; Γ; …)

At "asymptotically" high Ra:

$$Nu = 0.073Ra^{1/3}$$

Willem Malkus, Ed Speigel

$$Nu = CPr^{-1/4}\left[Ra/log(Ra)^3\right]^{1/2}$$

Bob Kraichnan

$$Nu = D\left[Ra^{3/2}ln(Ra)^{3/2}\right]^{1/5}$$

Jack Herring (*with input from Busse, Howard, Roberts, Stewartson, Malkus, etc…)*

Plausibility of scaling exponent 1/2

Convert gravitational potential energy into turbulent kinetic energy

$$\Delta\rho\, \mathrm{gH} \sim \rho u^2$$

$$u \sim \sqrt{\alpha\Delta\, \mathrm{TgH}}$$

From the equations for the mean temperature difference the transport of heat by turbulent fluctuations is

$$q' = \rho C_P \overline{u\theta}$$

involving the correlation between vertical velocity fluctuations and fluctuations in temperature which we take to be $\theta \sim \Delta T$

Assuming that the contribution of molecular transport can be ignored (essentially no diffusive boundary layers) we can set

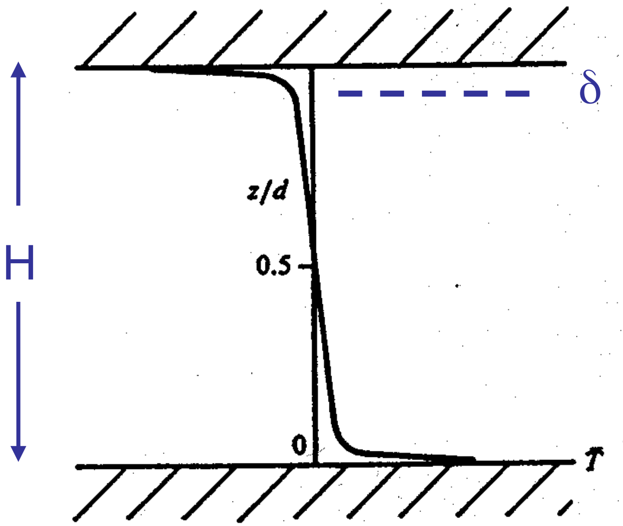
$$\rho C_P \overline{u\theta} \sim \mathrm{Nu} \cdot \frac{k\Delta T}{H}$$

Using the scales for vertical velocity and temperature fluctuations we then have

$$\mathrm{Nu} \sim \sqrt{\frac{g\alpha\Delta\, \mathrm{TH}^3}{\kappa^2}} = \left(\mathrm{RaPr}\right)^{1/2}$$

Plausibility of a 1/3 power law for Nu vs Ra (Malkus, Howard, Priestley)

$$\mathrm{Nu} = \frac{\mathrm{qH}}{k_f \, \Delta T}$$



At very high Ra the temperature gradient is **all at the wall**, across boundary layers of thickness δ

1. Heat flux across boundary layers: $q = \dfrac{k_f \, \Delta T}{2\,\delta}$

2. Rayleigh defined on the boundary layers:

$$\mathrm{Ra}_\delta = \frac{g\alpha \Delta T \delta^3}{\nu\kappa}$$

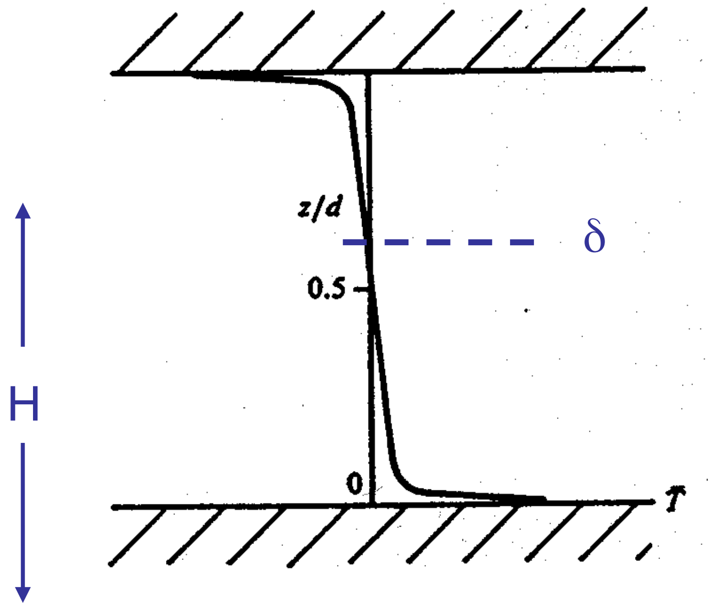
and assume that Ra_δ reaches marginal stability value Ra_c

3. Using (1) for q: Nu = ½ (H/δ)

4. Then from (2): $\delta = \left(\dfrac{2\,\mathrm{Ra}_c \, \nu\kappa}{g\alpha \Delta T} \right)^{1/3}$

5. Substituting δ from (4) into (3) and using (2):

$\mathrm{Nu} = [\mathrm{Ra}/(16\mathrm{Ra}_c)]^{1/3} \sim C\mathrm{Ra}^{1/3}$



One caveat however ….

However, from Howard (1962): The main point is that the stability of a boundary layer at the bottom of a semi-infinite region, which is essentially our problem for large R, is really rather different from the ordinary case of a finite layer. The critical Rayleigh number for *marginal* stability, based on the boundary layer thickness, is in fact zero — such layers in an infinite region are always unstable.

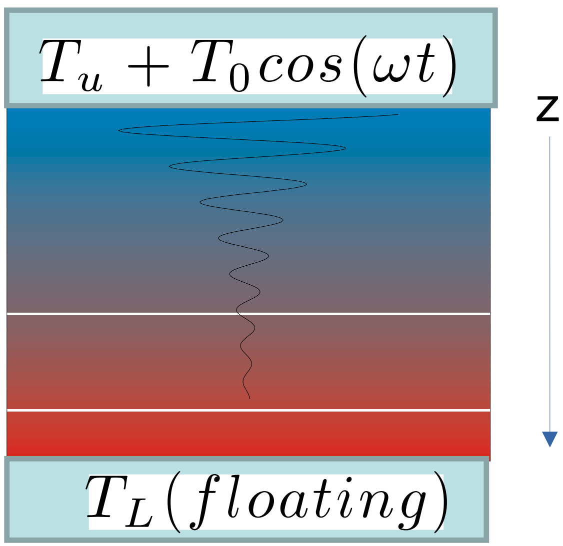
VOLUME 57, NUMBER 5 PHYSICAL REVIEW LETTERS 4 AUGUST 1986

Instability of a Thermal Stokes Layer

Joseph J. Niemela and Russell J. Donnelly
Department of Physics, University of Oregon, Eugene, Oregon 97403
(Received 21 March 1986)

We examine experimentally the stability of a Stokes layer in a fluid near a boundary whose temperature is modulated as $T_0 \cos\omega t$. We define an appropriate Rayleigh number for the problem and determine its critical value. Increased stabilization is observed to accompany a reduction in the Prandtl number. We observe hysteresis effects near the critical Rayleigh number, including a double hysteresis loop, which appear qualitatively similar to recent predictions of Roppo, Davis, and Rosenblat.

PACS numbers: 47.20.Bp, 47.25.Qv



Diffusion eqn* (1D): $\dfrac{\partial T}{\partial t} = \kappa \dfrac{\partial^2 T}{\partial z^2}$

Solution:

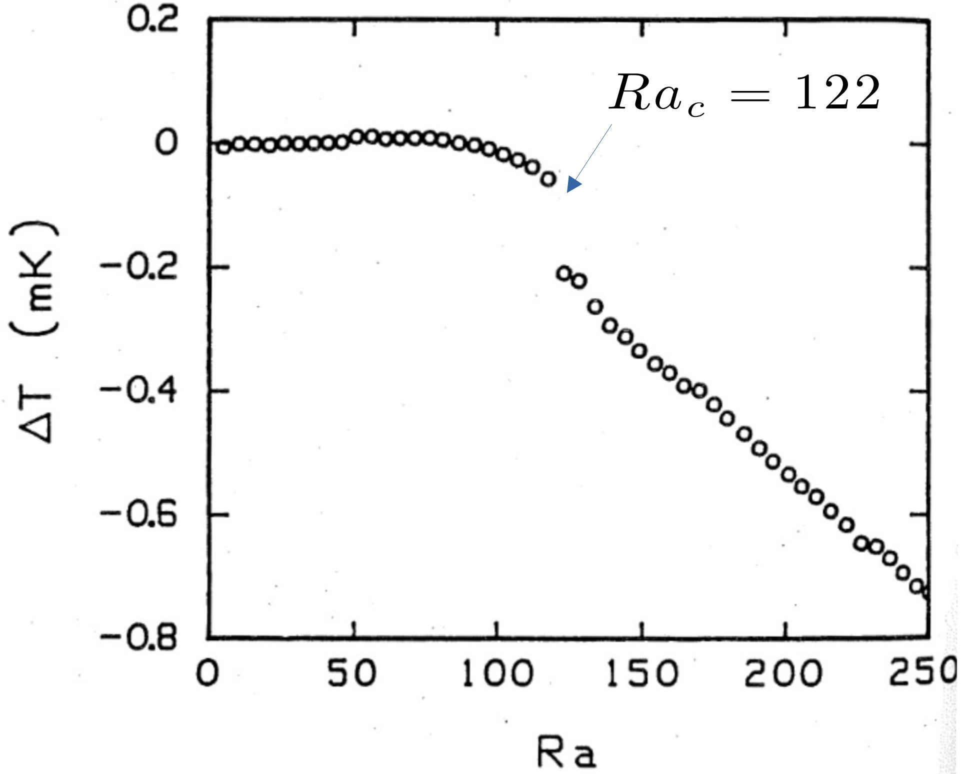
$$T(z,t) = T_0 exp\left(\frac{-z}{\delta}\right) cos(\omega t - z/\delta)$$

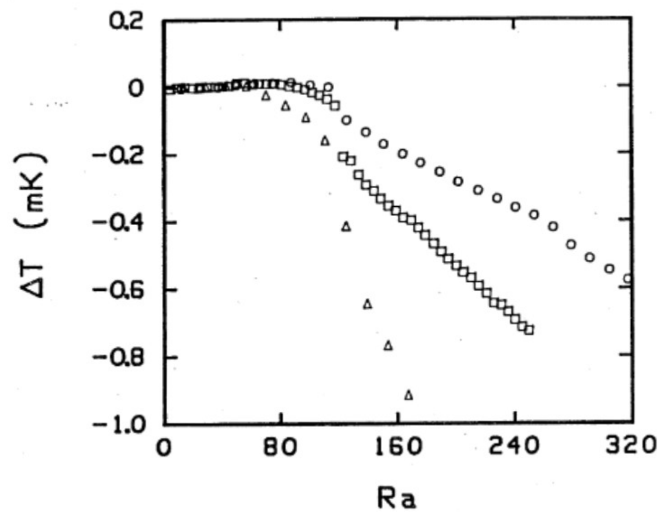
$$\delta = \sqrt{\frac{2\kappa}{\omega}}$$ Stokes layer

$$Ra_\delta = \frac{g\alpha T_0 \delta^3}{\nu\kappa}$$

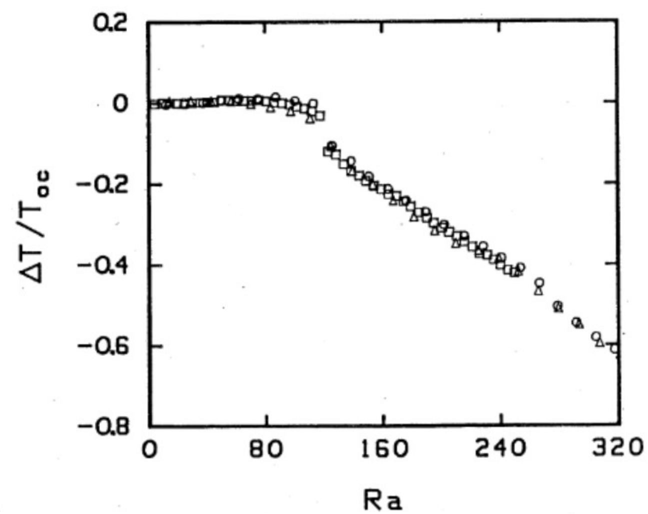
An inverse Nu-1: $\Delta T \equiv \langle T_L(t)\rangle - \langle T_u(t)\rangle = 0$ in absence of convection

Marginally stable boundary layers!





Independent of b.l. thickness



Gershuni G.Z., & Zhukhovitskii, E.M., 1976 *Convective instability of incompressible fluids* Keter, Jerusalem

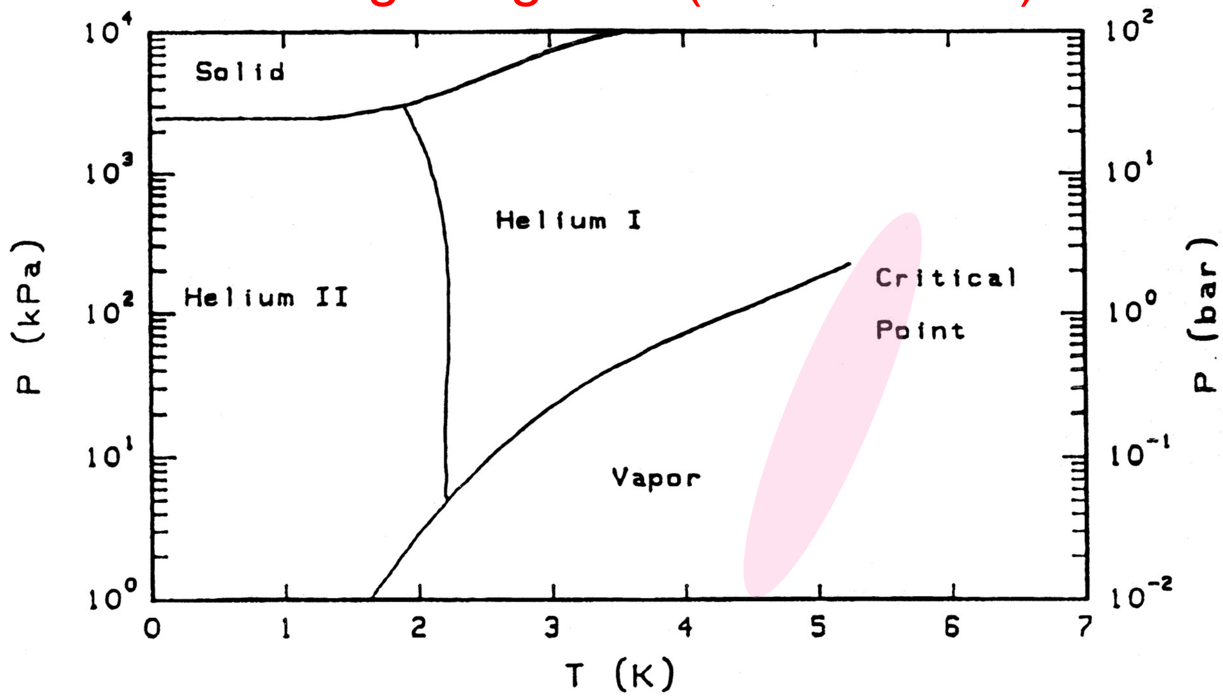
Hall. P. 1985 "Instability of time-periodic flows" NASA contract report no. 178009 & private communication

J. B. Swift and P. C. Hohenberg, 1987 "Modulated convection at high frequencies and large modulation amplitudes" Phys. Rev. A **36**, 4870

This Ra_c is an order of magnitude smaller than for normal fluid layers ~ 10^3.

Gives a coefficient of **0.08** in our simple argument for 1/3 scaling -we'll see this later

How to get high Ra (and lots of it)



$$\mathrm{Ra} = g \cdot \left(\frac{\alpha}{\nu\kappa} \right) \cdot \Delta T \cdot H^3$$

cgs units

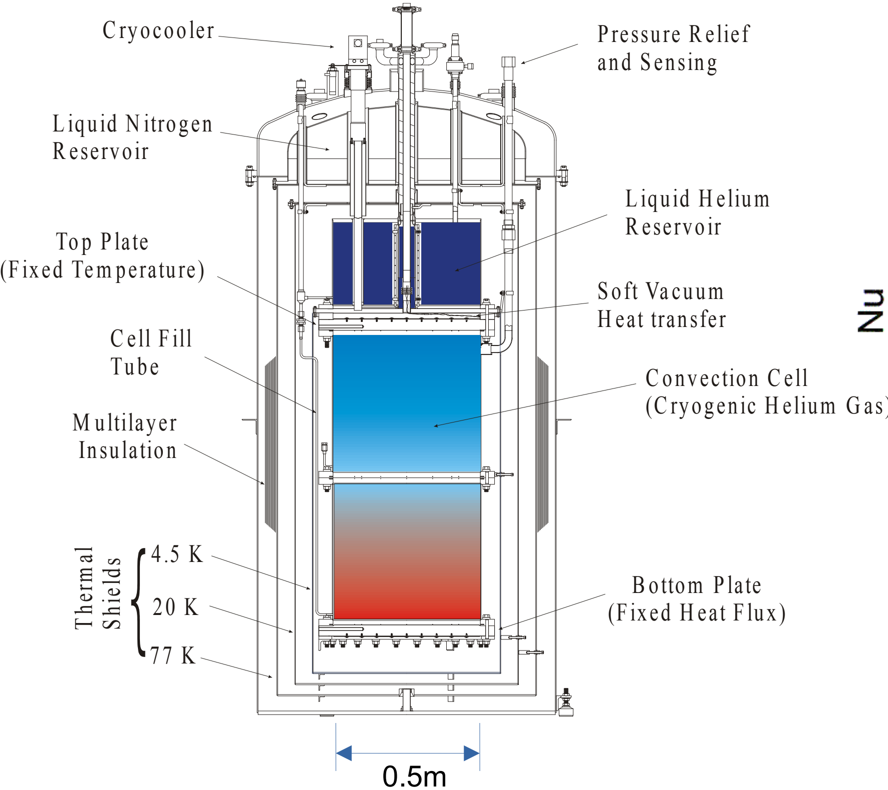
4.4 K , 2 mbar:
$\alpha / \nu\kappa = 5.8 \times 10^{-3}$

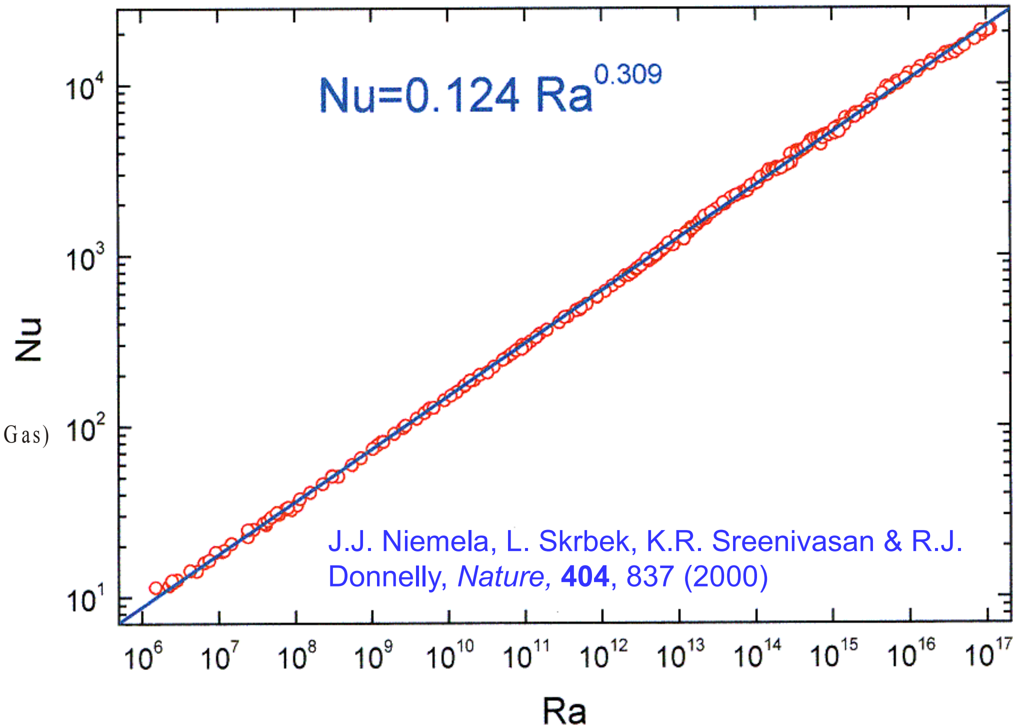
5.25 K, 2.4 bar:
$\alpha / \nu\kappa = 6.5 \times 10^{9}$

Ra ~ ($\rho^2 \alpha C_P$). Ra increases as ρ^2 away from critical point and as αC_P in its vicinity

12 orders of magnitude of Ra, all in turbulent regime (scaling) for H sufficiently large

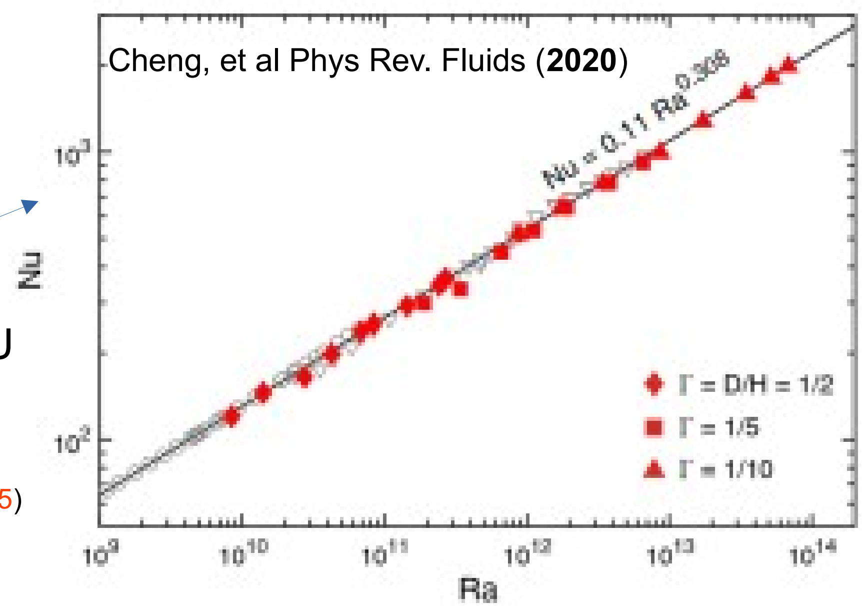
$\Gamma = 0.5$

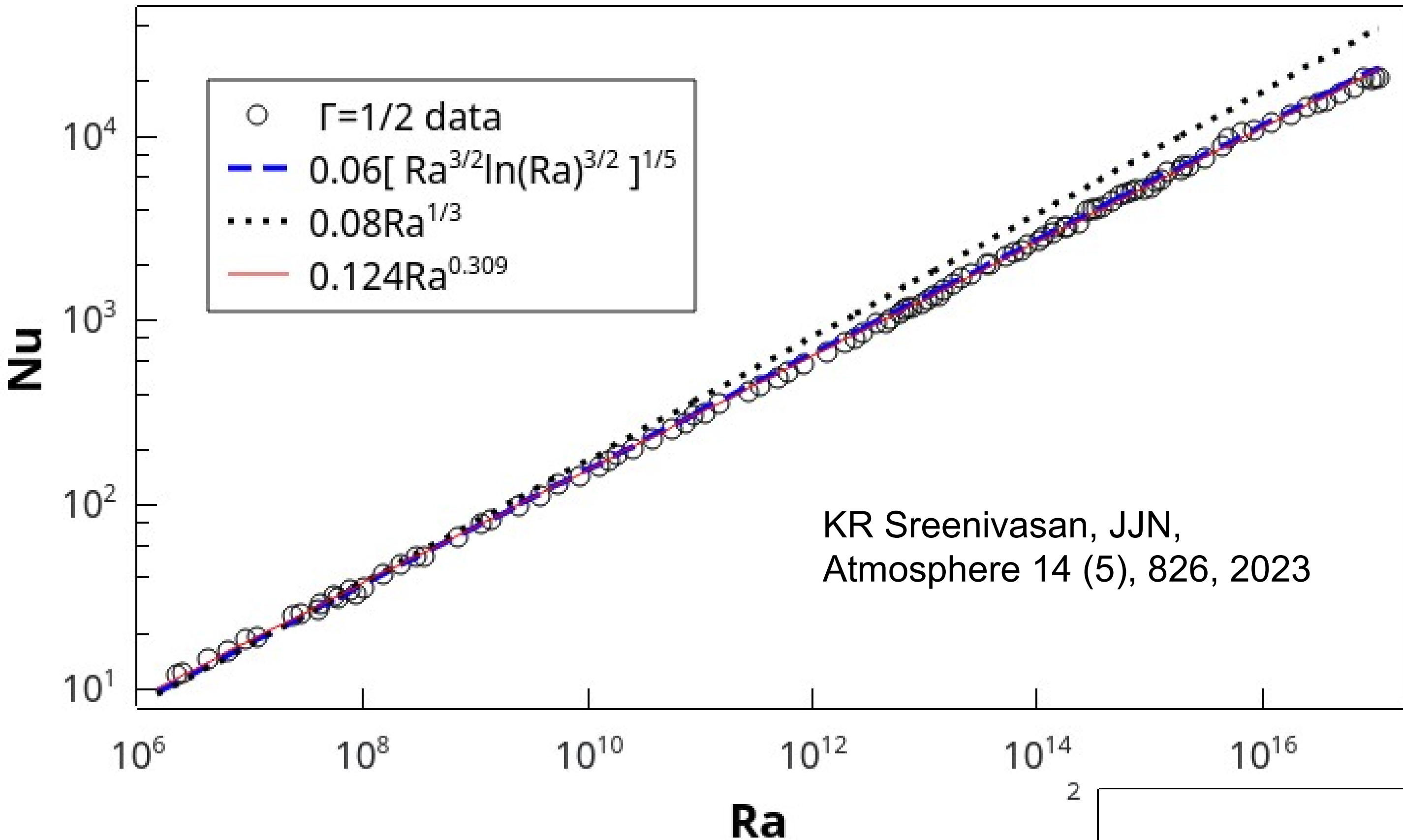




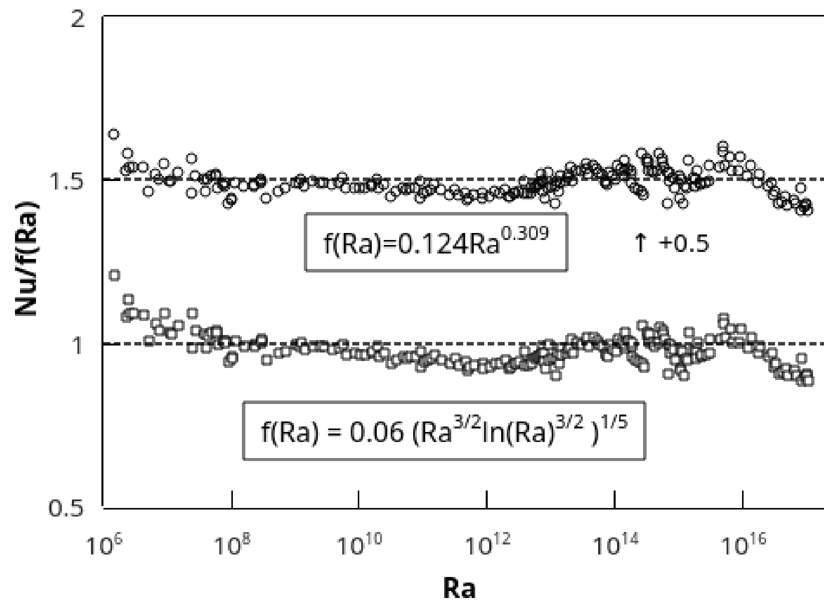
Recently:
4 meter (*max*) water facility at TU Eindhoven (Γ **= 0.5, 0.2, 0.1**)

$Nu = 0.11(+-0.02)\ Ra^{0.308(+-0.005)}$





Using the **Stokes layer marginal stability value** for Ra_c we have Nu= $0.08Ra^{1/3}$ shown in the plot above



What turbulent convection looks like:

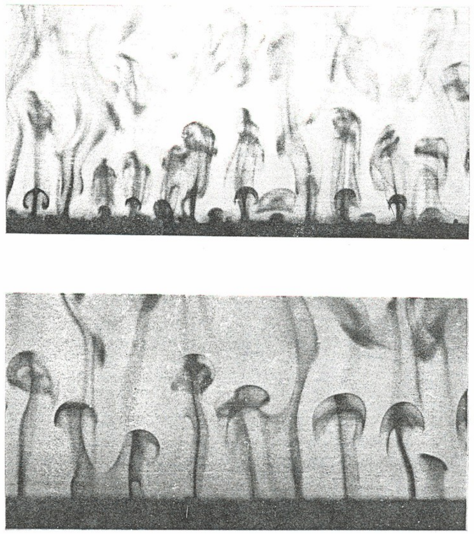
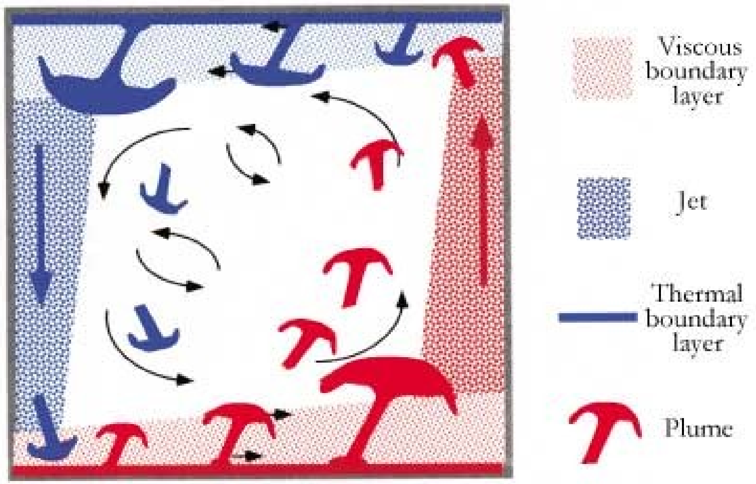


FIGURE 1. Photographs of thermals rising from a heated horizontal surface.



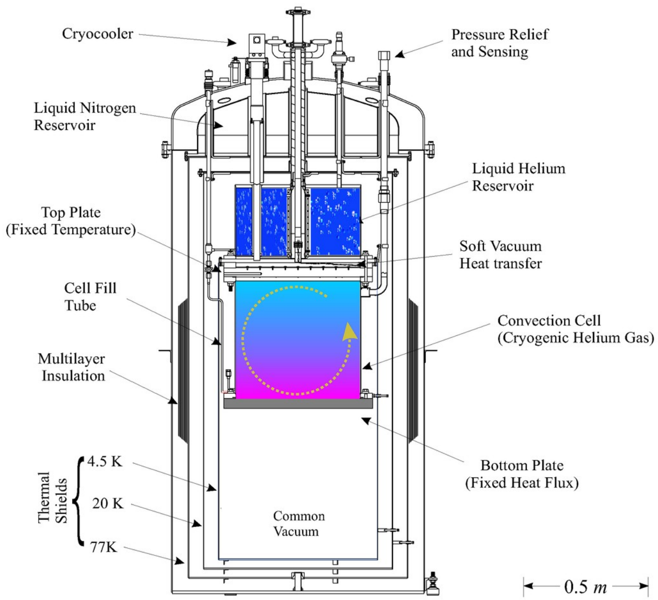
After L. kadanoff, Physics Today 2001

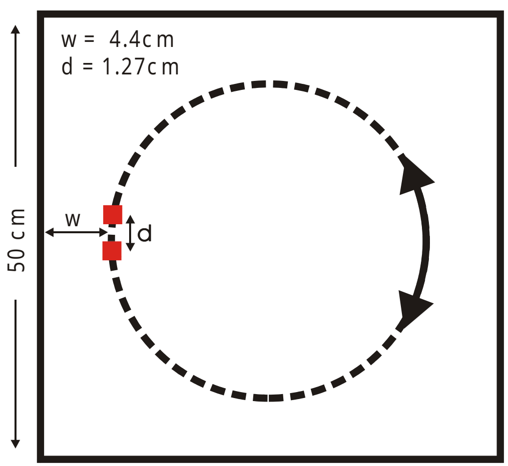
Herring: "*The physical picture of free boundary convective process predicted by the model is that of a large-scale motion dominating the central region between the conducting plates. This large-scale motion sweeps with it the temperature fluctuation field whose main variances occur in a thin boundary layer of vertical extent 1/Nu. The horizontal scale of both the dominant motion and the temperature fluctuation field is comparable to the distance between the conducting plates*."

This describes well the experimental observations with rigid boundaries

The Biot number associated with copper plates at low temperatures is very low: *Ideal* plume production is not impeded by the apparatus

An aspect ratio unity cell for maximizing the mean wind

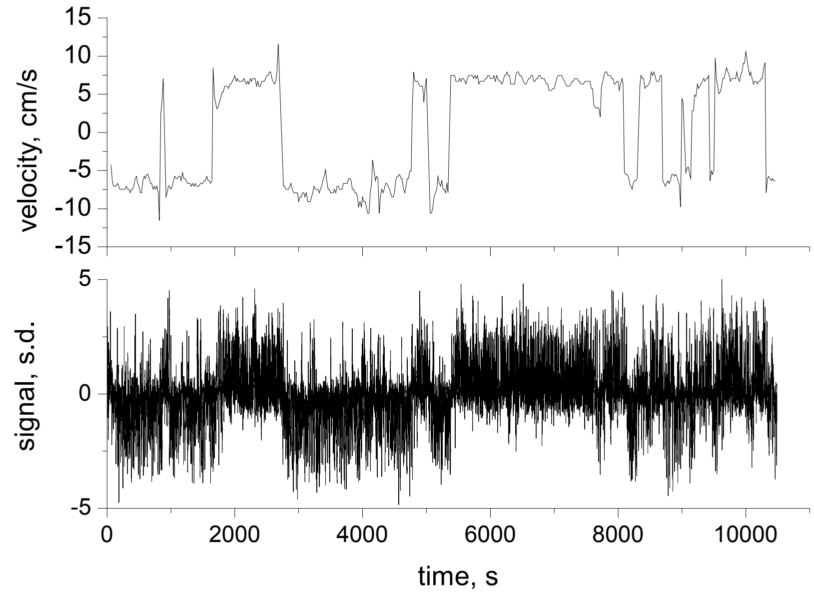




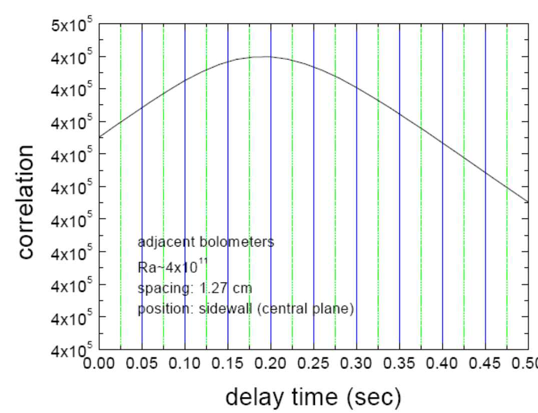
250-micrometer NTD-doped Ge sensors are placed in various positions in the flow.

Stabilization: 10^5 turn-over times of the wind

Max run times: 10^4 turn-over times of the wind

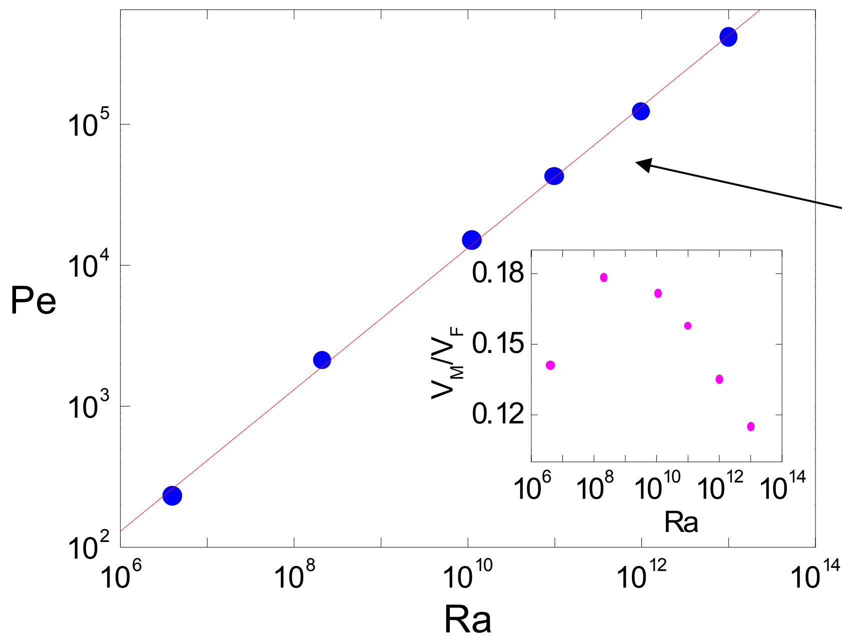


Maximizing the correlation between temperature signals gives the magnitude and direction of a large scale circulation.

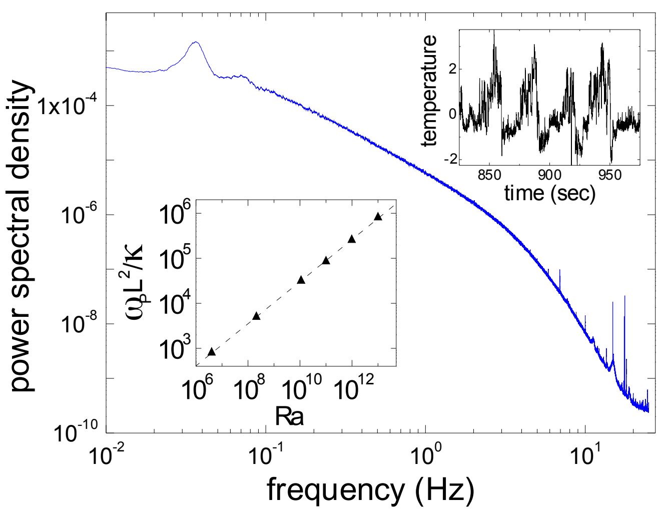


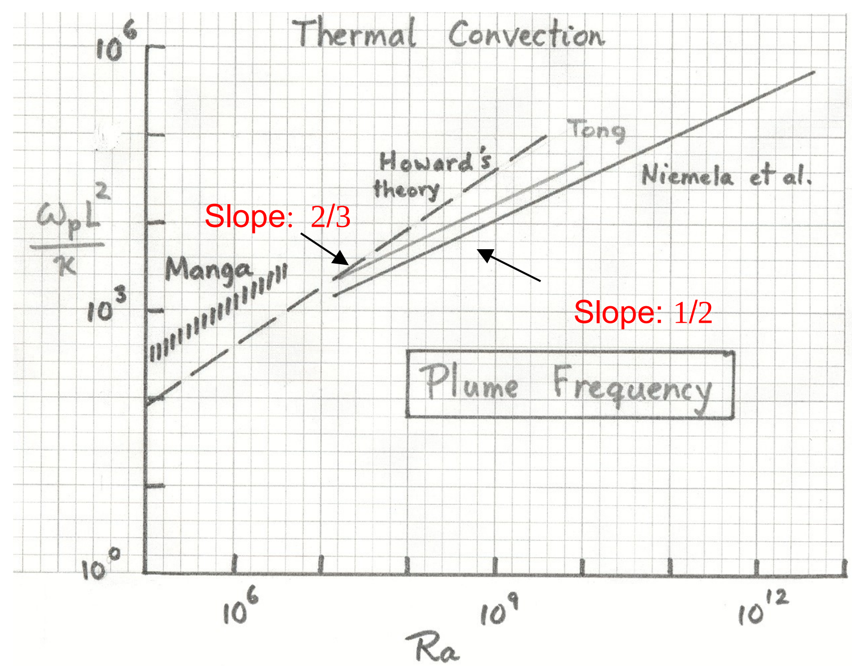
Measurement of the large scale circulation

$$Pe = Re \cdot Pr = \frac{V_M H}{\kappa}$$

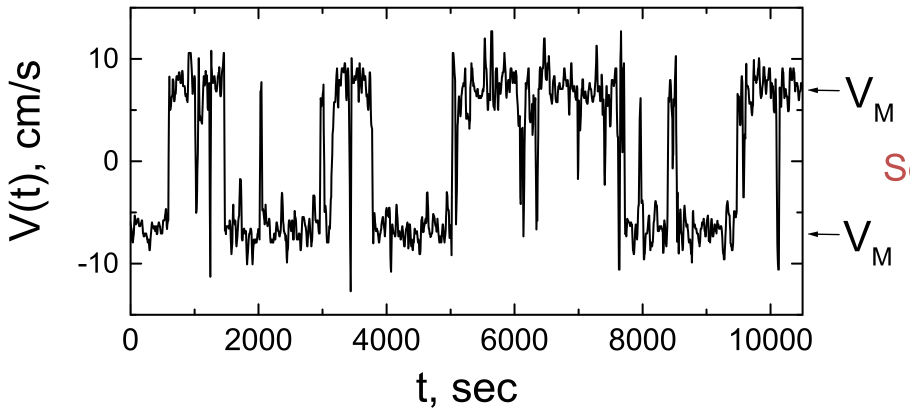


$$V_m\left(\frac{H}{\kappa}\right) = 0.13Ra^{1/2}$$



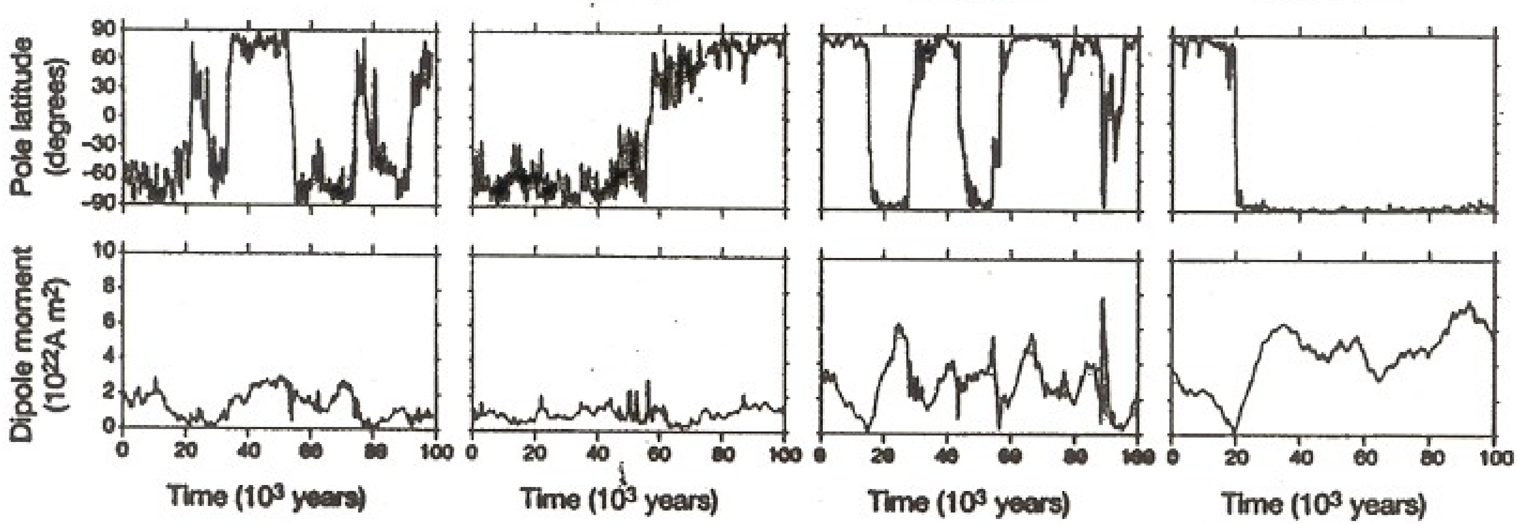


The mean wind and its reversals

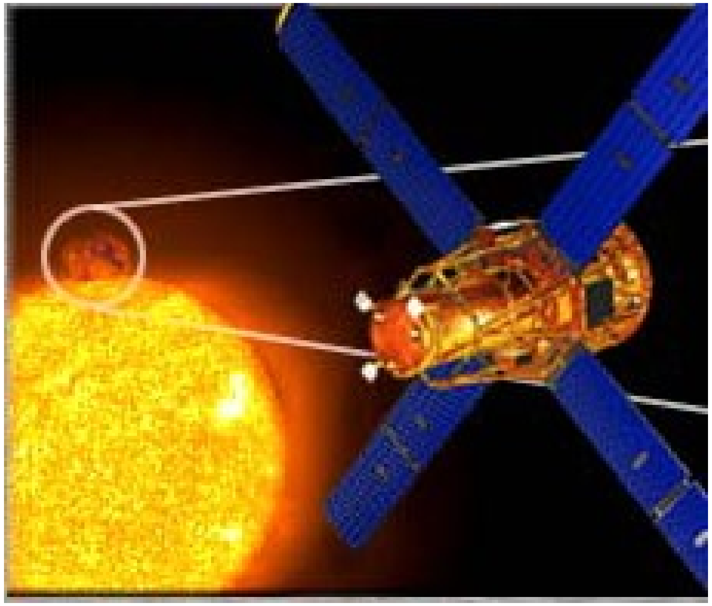


Segment of continuous 5.5-day time series.

Geomagnetic polarity reversals: range of time scales~ 10^3-10^5 years.



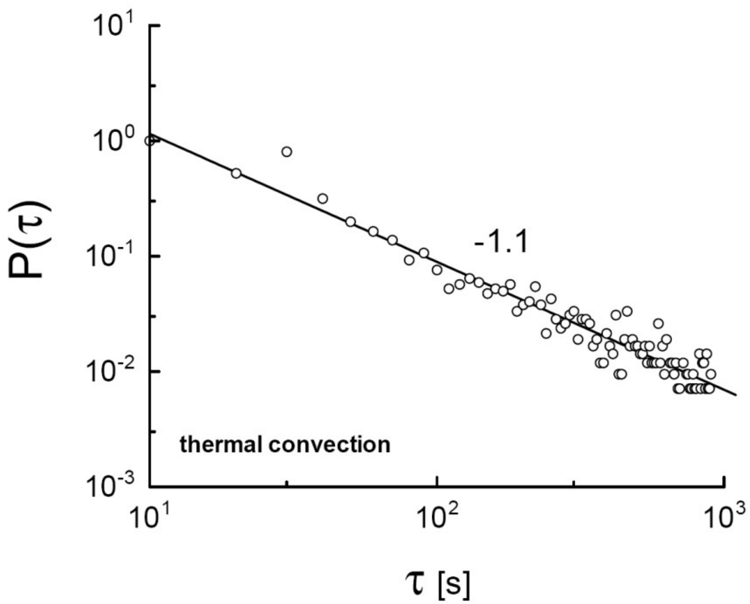
Glatzmaier, Coe, Hongre and Roberts *Nature* **401**, p. 885-890, 1999

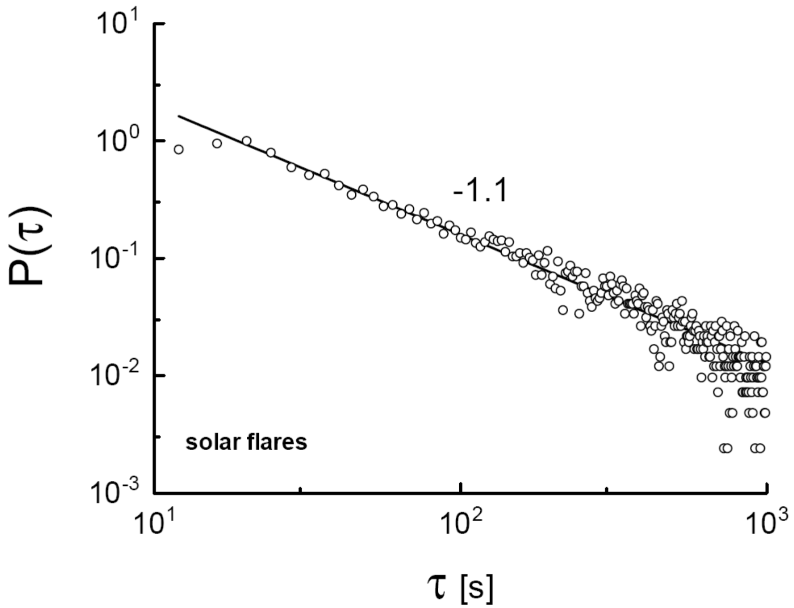


Medium energy solar flares owe their duration to turbulent convective motions in the convective zone of the sun which shuffle footprints of the magnetic coronal loops (Parker, 1994).

Reuven Ramaty High Energy Solar Spectroscopic Imager (RHESSI)

Comparison of the duration of single-direction wind in RBC experiments to the duration of solar flares observed by RHESSI





Turbulent heat transfer II

advection-diffusion equation

$$\frac{\partial T}{\partial t} + U_j \frac{\partial T}{\partial x_j} = \kappa \frac{\partial^2 T}{\partial x_j \partial x_j}$$

Decomposition and averaging over fluctuations yields for the vertical heat flux

$$q_3 = \rho c_P \left(\overline{\theta u_3} - \kappa \frac{\partial \overline{T}}{\partial x_3} \right)$$

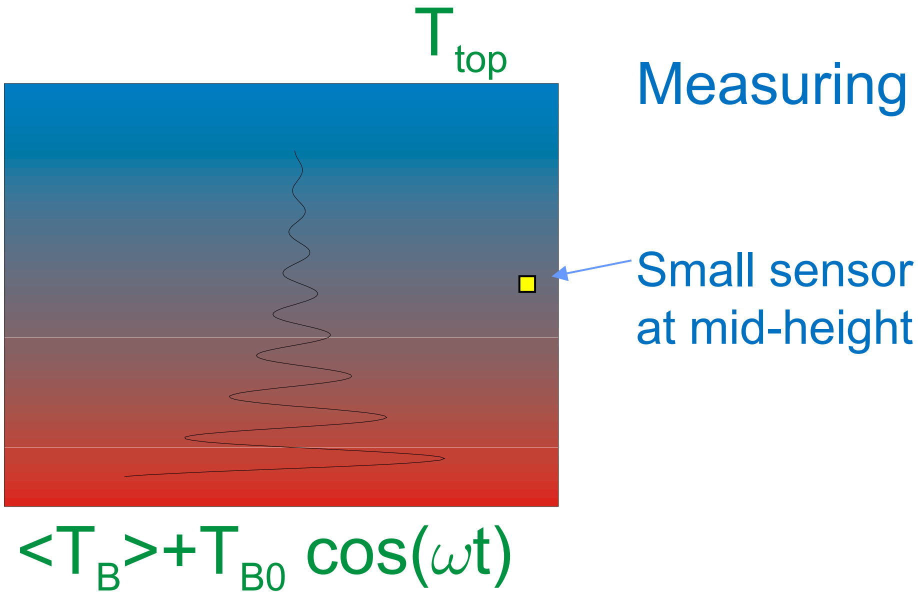
$$\overline{\theta u_3} \equiv - \kappa_T \frac{\partial \overline{T}}{\partial x_3}$$

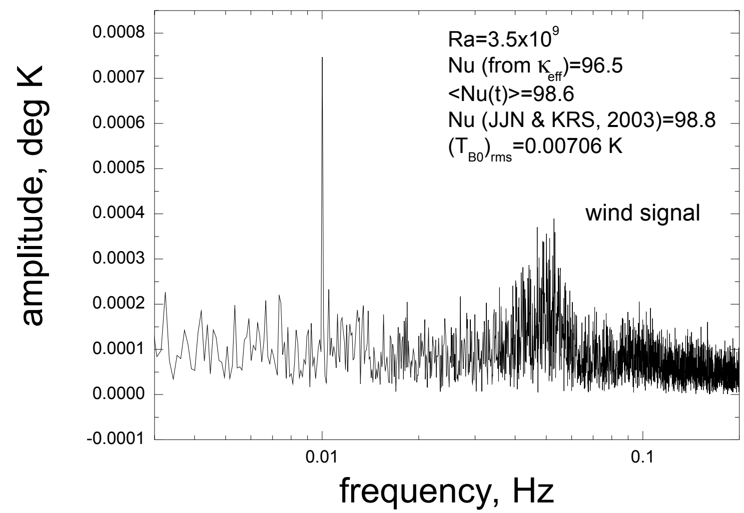
a *convenient definition* treating turbulence as a diffusive "fluid"

$$\boxed{Nu = \frac{\kappa^{eff}}{\kappa}} \qquad \kappa^{eff} \equiv (\kappa + \kappa_T)$$

,

Measuring an "effective thermal diffusivity"



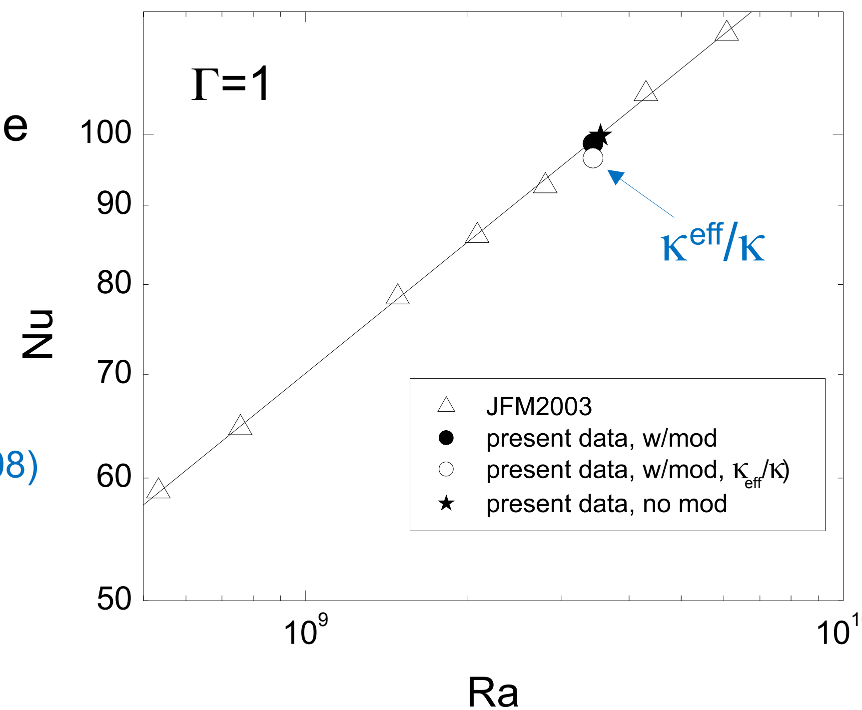


Solution to diffusion equation: $\dfrac{\partial T}{\partial t} = \kappa^{eff}\dfrac{\partial^2 T}{\partial z^2}$

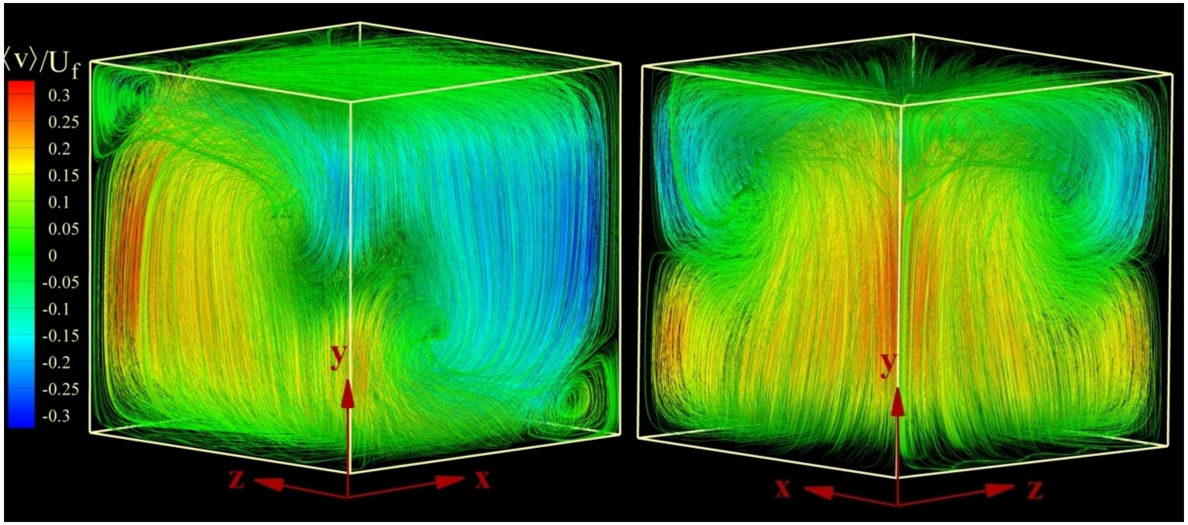
$T(z,\omega) = (T_{B0})_{rms}\exp(-z/\delta_S)$ for the amplitude

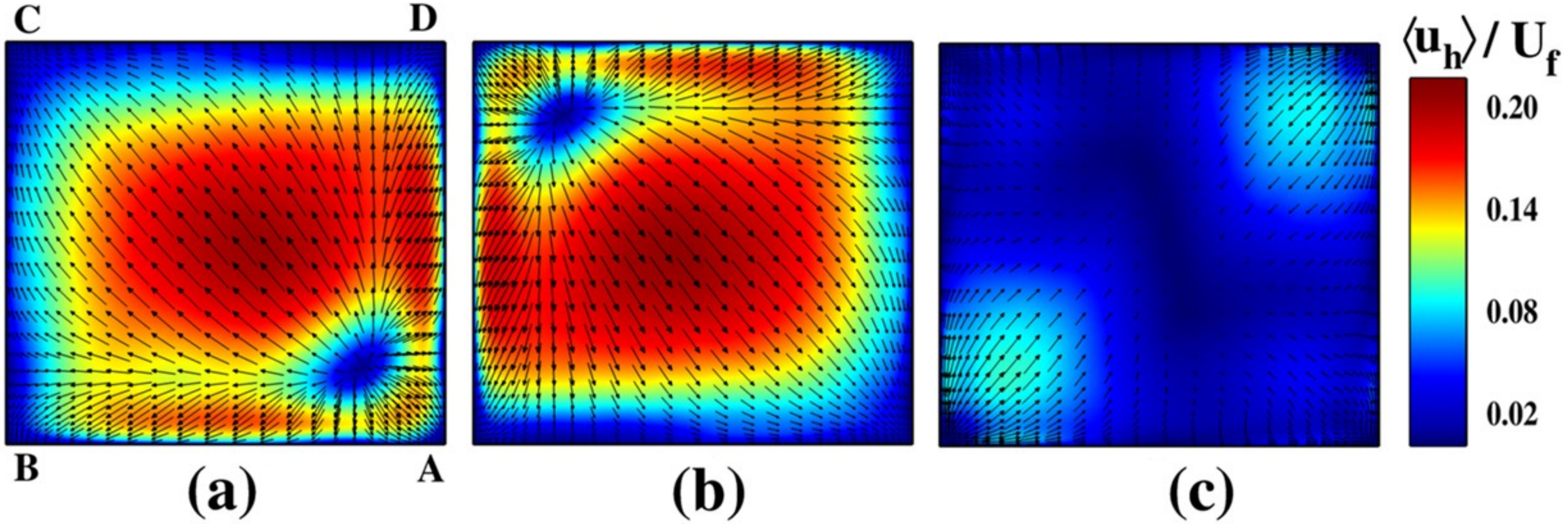
$$\delta_S = \sqrt{\frac{2\kappa^{eff}}{\omega}}$$

JJN, K.R. Sreenivasan, Phys Rev. Lett. . **100,** 184502 (2008)



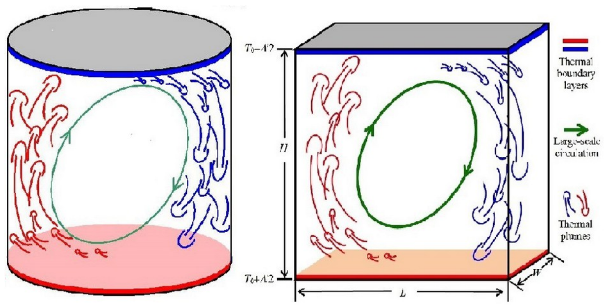
Simulations in a Cube ($Ra=10^8$)





N. Foroozani, JJN, V. Armenio, and K. R. Sreenivasan, Phys. Rev. E **90**, 063003 (2014)
N Foroozani, JJN, V Armenio, KR Sreenivasan, Physical Review E **95** (3), 033107 (2017)

Application to Daya and Ecke's question: Does container shape affect rms statistics in the bulk? (Daya and Ecke [*Phys. Rev. Lett.* **87**, 184501 (2001))



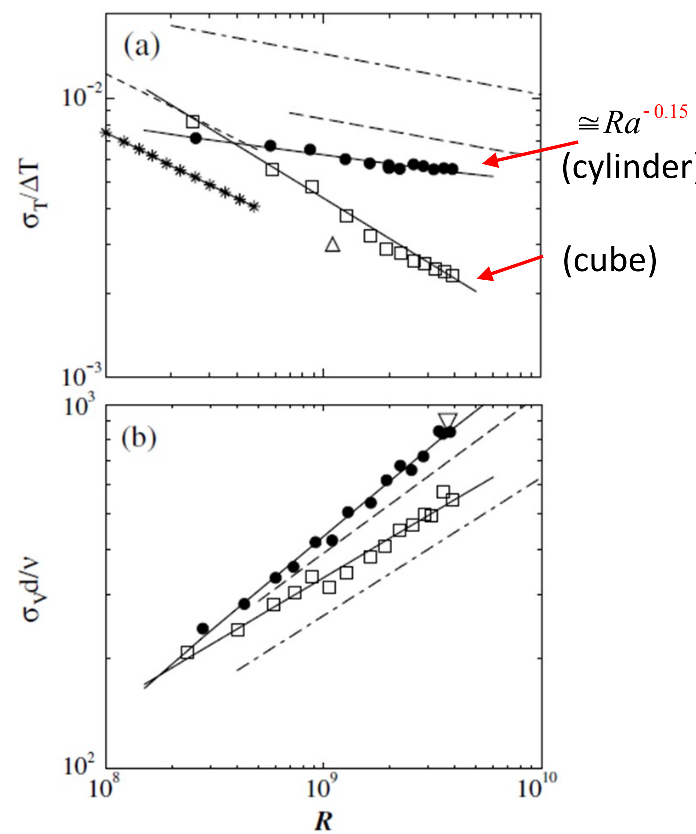
Daya and Ecke found in **cube**:

$$\frac{\sigma_T}{\Delta T} \propto Ra^{-0.48 \pm 0.03} \qquad \frac{\sigma_V H}{\nu} \propto Ra^{-0.36 \pm 0.05}$$

Our results in a cube:

$$\frac{\sigma_\rho}{\Delta\rho} = 0.59 Ra^{-0.46} \qquad \frac{\sigma_V H}{\nu} = 0.32 Ra^{0.39}$$

N. Foroozani, JJN, V. Armenio, and K. R. Sreenivasan, Phys. Rev. E **90**, 063003 (2014).

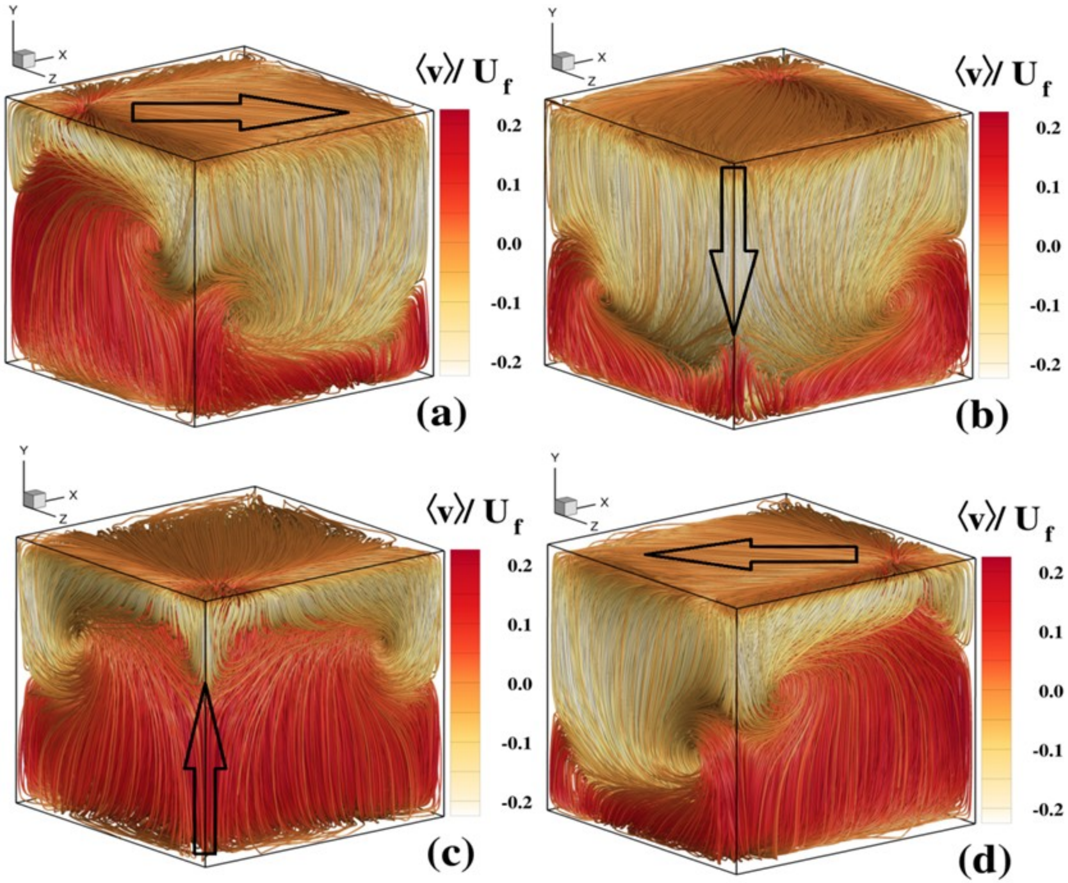


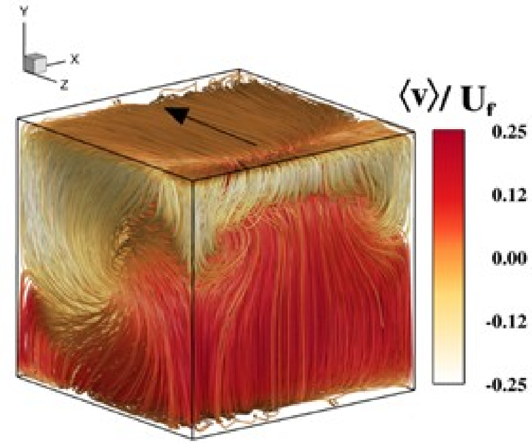
From Daya and Ecke 2001

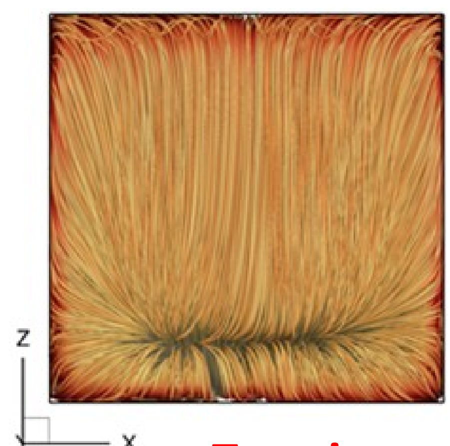
The clue is shown in the simulations for a cube!

Re-orientations of the Large Scale Flow in a Cube, Ra=10^8

Transient states in between, parallel to side wall





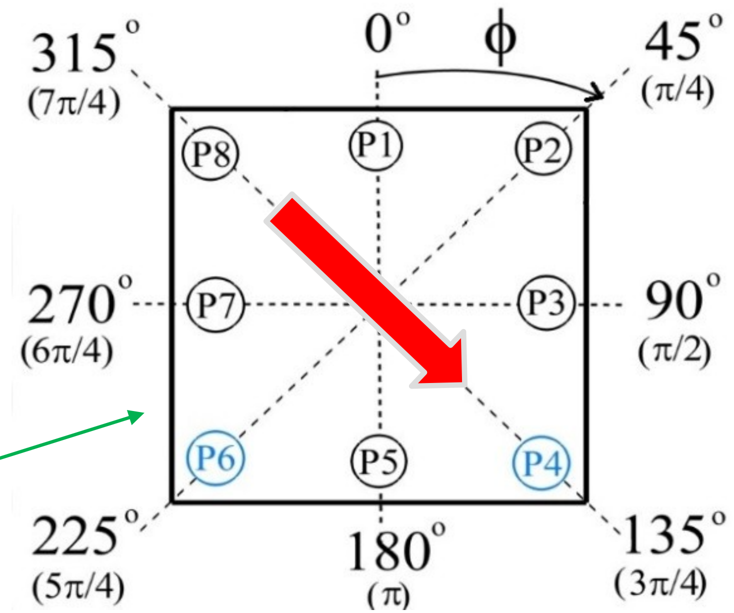


Top view

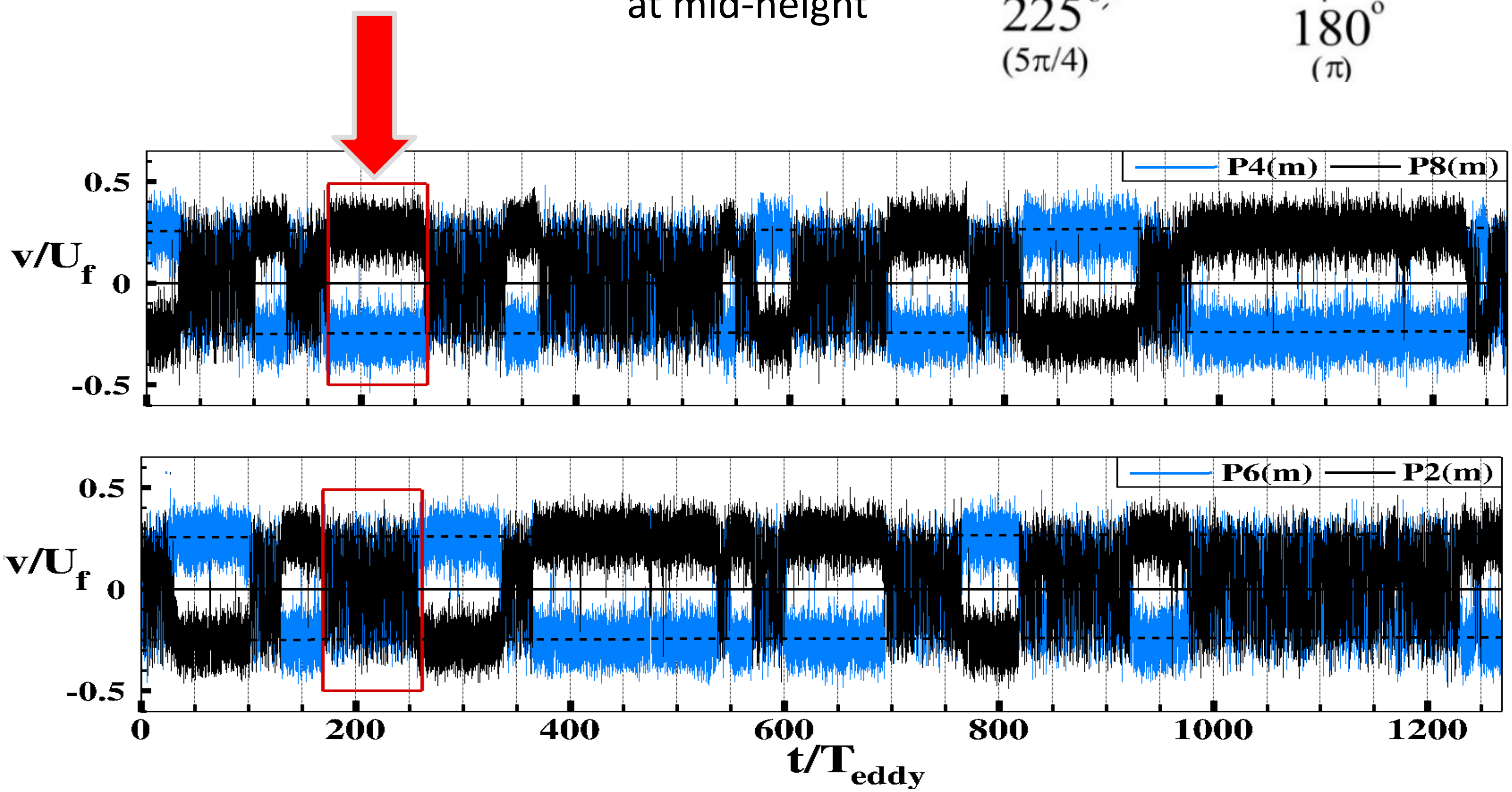
8 discrete flow states in total

N. Foroozani, J. J. Niemela, V. Armenio, and K. R. Sreenivasan Phys. Rev. E 95, 033107 (2017)

What signals would look like to experiments....

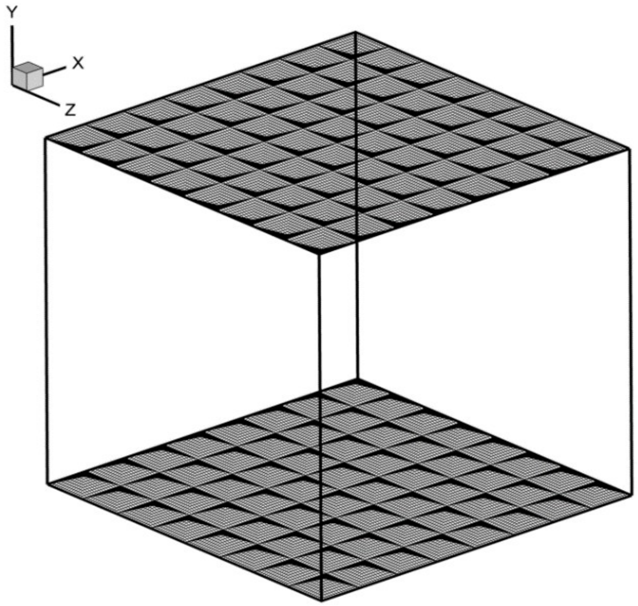


Horizontal plane
at mid-height

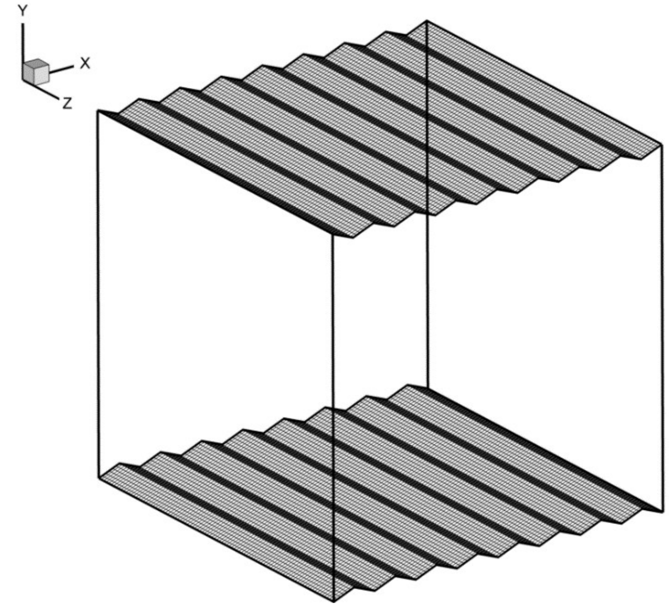


Adding “2D” and 3D roughness elements

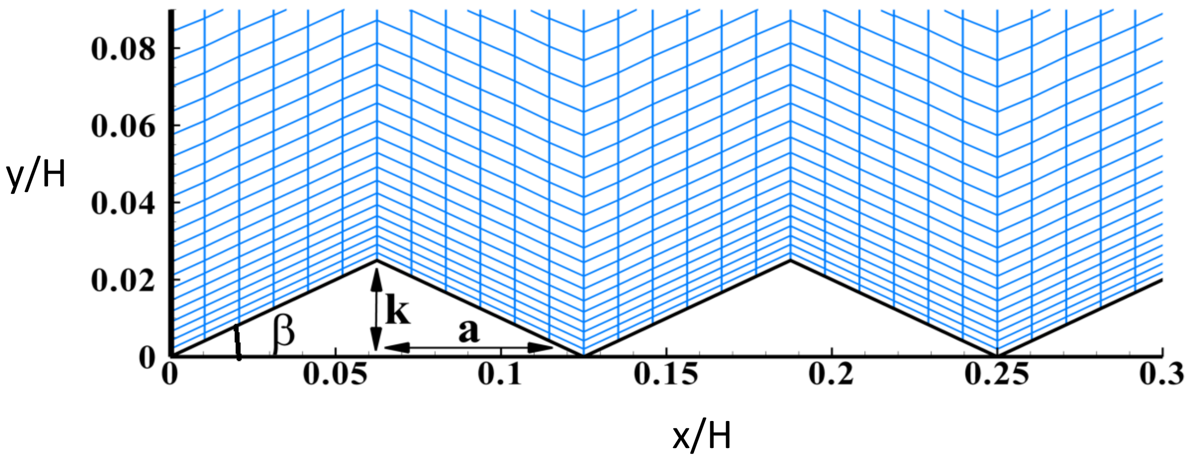
Pyramids (64)



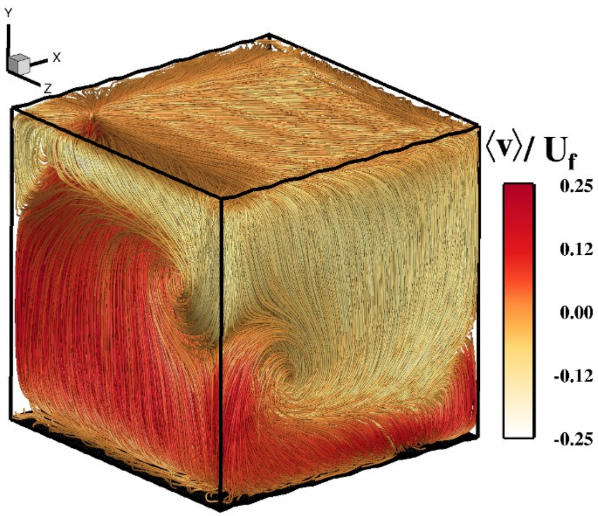
Grooves (8) parallel to side wall!



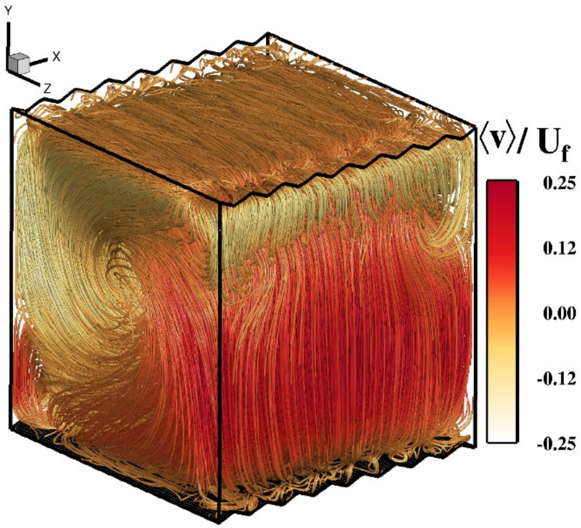
Grid mesh:



Grooves

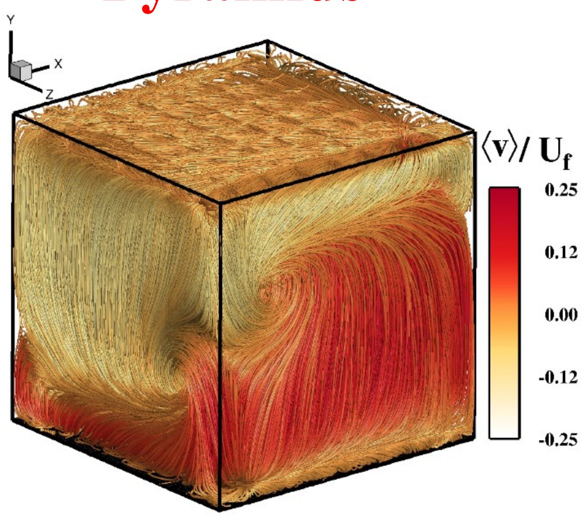


Hydrodynamically smooth



Hydrodynamically rough

Pyramids



Hydrodynamically rough (same configuration as for smooth)

Facilities located at Elettra Synchrotron Laboratory, Trieste