Magnetohydrodynamic Two-Phase Slip Flow and Heat Transfer of Dusty Tangent Hyperbolic Fluid over an Expansive Porous Sheet

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Introduction Cont'd

- **What is a Fluid?**
- **Fluid**: A substance in the liquid or gas phase.
- *A fluid deforms continuously under the influence of a shear stress*, no matter how small.
- In solids, stress is proportional to *strain*, but in fluids, stress is proportional to *strain rate*.




Introduction Cont'd

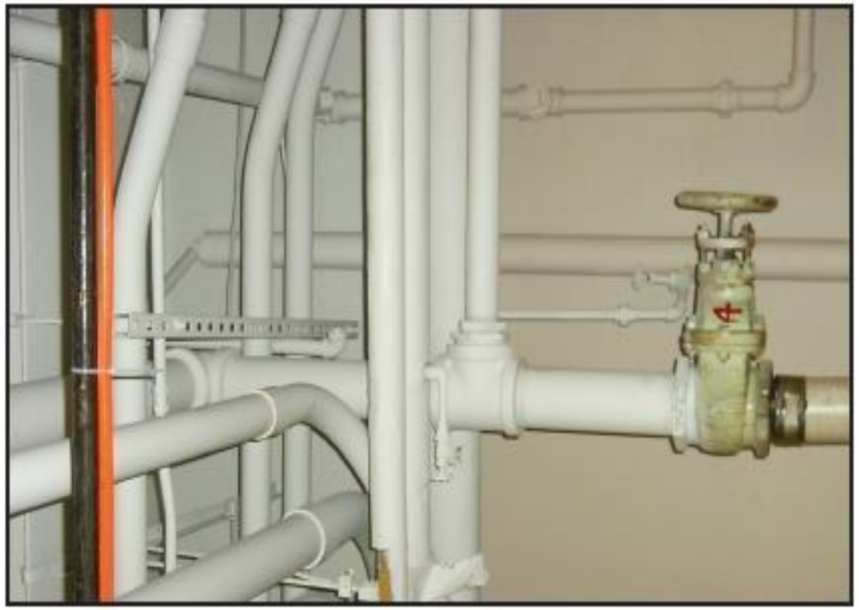
- Fluids can be categorized as Newtonian and non-Newtonian.
- Non-Newtonian fluid models: Tangent hyperbolic fluids, Casson, Williamson, tangent hyperbolic fluids, etc.
- Tangent hyperbolic fluid exhibits a shear thinning attributes, e. g. ketchup, paints, biological fluids, etc.
- Crucial applications of this fluid are found in polymer industries, paint industries, food processing & manufacturing.

Introduction Cont’d

- Dusty fluids are two-phase fluid flows (also particle-laden flows) that consist of suspended dust particles or related solid items distributed in a fluid.
- They particles can be of various compositions and sizes, as small as fine particulate matter or even rubble. Common in construction and industrial sites, mudflows, sediment-laden rivers, etc.
- Fluids containing suspended dust particles can greatly influence flow dynamics, heat and mass transfer, and other thermophysical properties.
- Such as petroleum and crude oil engineering, environmental and geotechnical engineering (e.g., pollution control etc.), powder technology (e.g., fluid processing, drug and pharmaceuticals, etc.).

Some Application Areas of Fluids & Heat Transfer in Technology

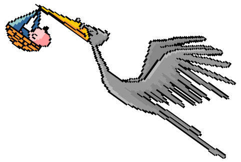
Wind turbines



Piping and plumbing systems

Application Areas of Fluid Mechanics and Heat Transfer inTechnology

Fluid dynamics and heat transfer is used extensively in the design of artificial hearts



Natural flows and weather

Power plants

More Applications

- **Internal Combustion Engines- In Transportation Systems**
- **Thermotherapy**

(The use of heat in Therapy)

Boats

Aircraft and spacecraft

Human body

Cars

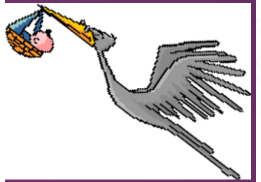
Mathematical Modelling of Physical Problems

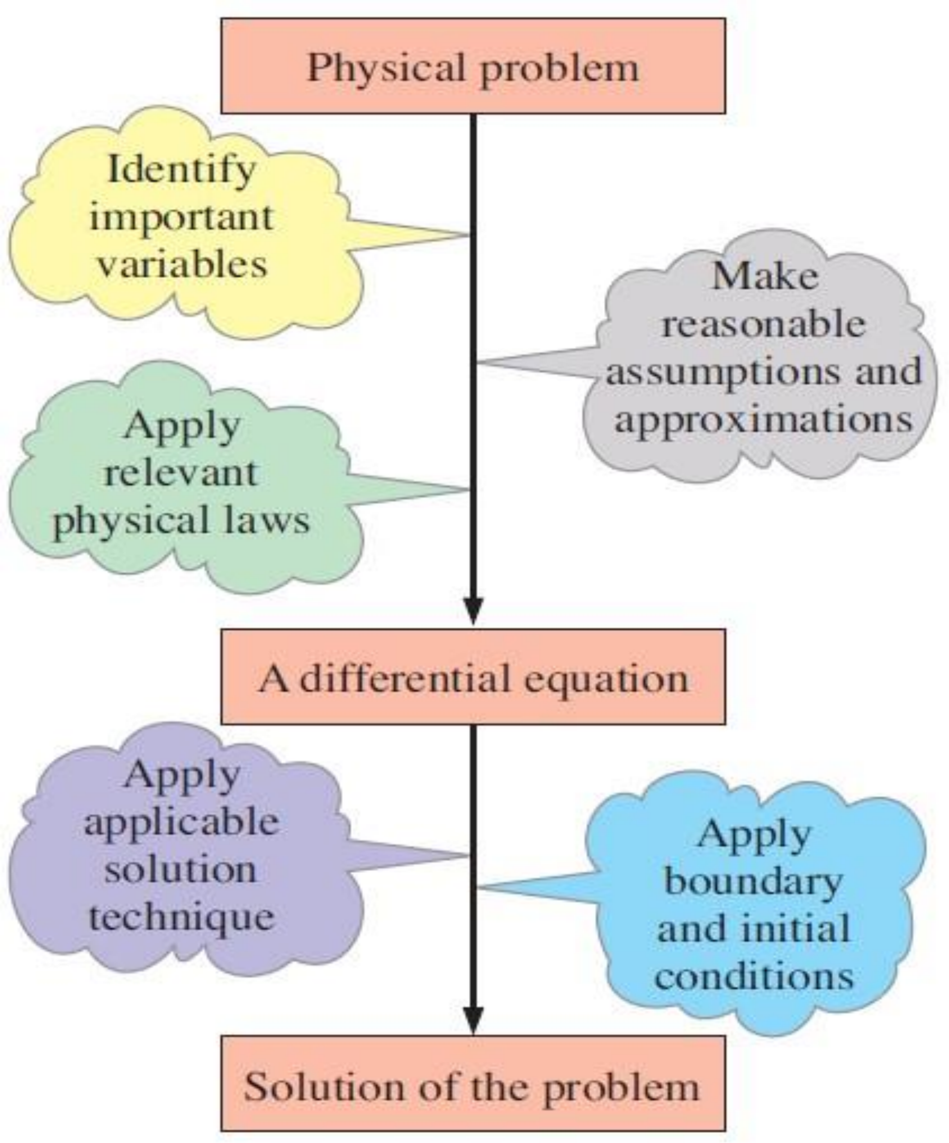
Why do we need differential equations?

Most scientific problems involve eqns that relate the changes in some key variables to each other.

differential equations provide precise mathematical formulations for the physical principles and laws by representing the rates of change as ***derivatives*.**

They are used to investigate a wide variety of problems in sciences and engineering.





Basic Assumptions :

- The flow is steady, incompressible on a 2D sheet
- The Fluid is a tangent hyperbolic fluid.
- The radiative heat flux term in x direction is negligible as compared to that in the y direction.
- Joule heating and Viscous dissipation are included
- The thermal conductivity is temperature-dependent.
- The sheet temperature is aided through a convective heating mechanism from a hot fluid with a temperature T_f

The Flow Geometry

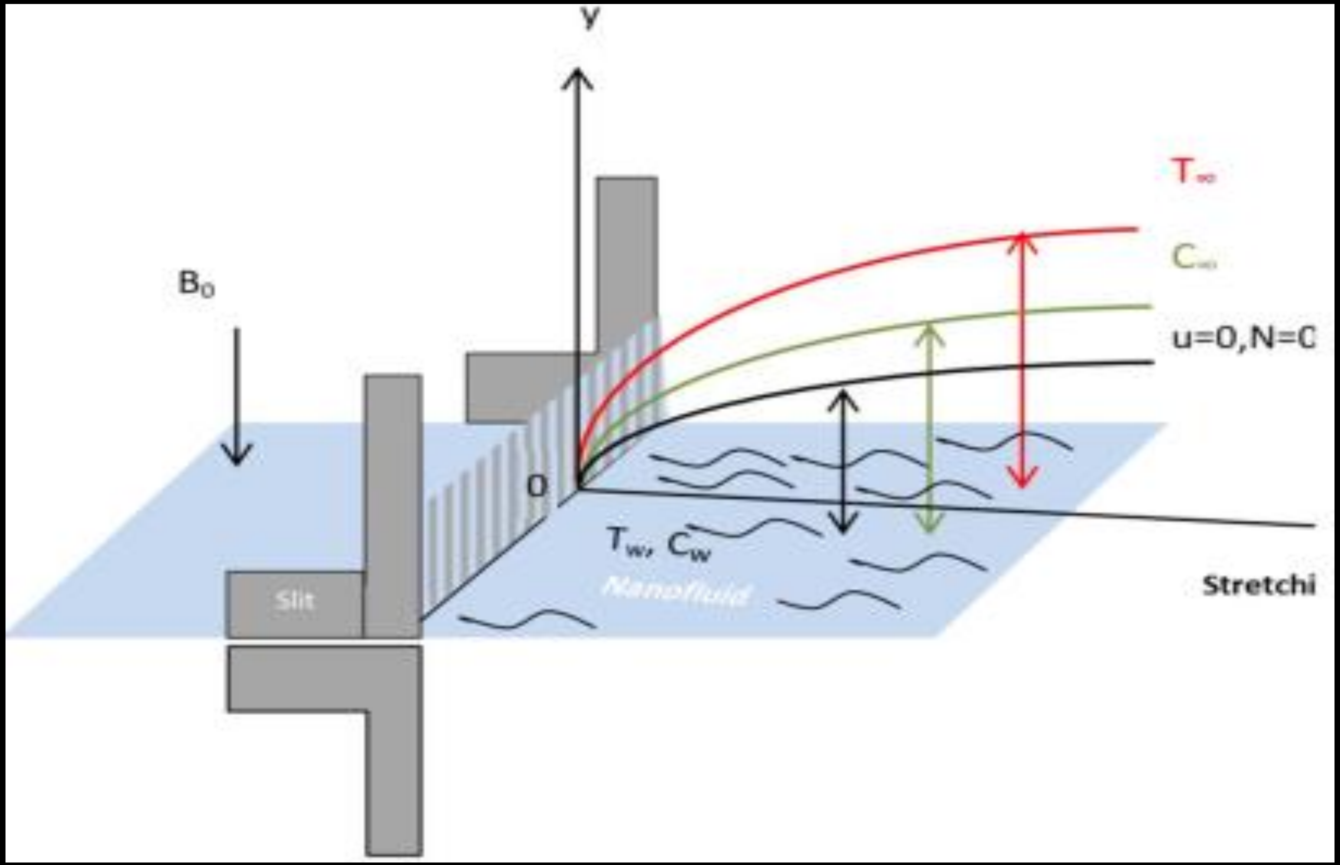


Figure . 1: ***Cartesian system & Physical model***

The Governing Equations

For fluid phase:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{\mu}{\rho}(1-\kappa)\frac{\partial^2 u}{\partial y^2}+\frac{\mu}{\rho}\sqrt{2}\kappa\Gamma\frac{\partial^2 u}{\partial y^2}\frac{\partial u}{\partial y}-\frac{\sigma B_0^2}{\rho}u+\frac{\mu}{\rho K_p}u+\frac{KN}{\rho}\left(u_p-u\right), \tag{2}$$

$$\begin{aligned}(\rho c_p)\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\mu\left((1-\kappa)\left(\frac{\partial u}{\partial y}\right)^2+\frac{\Gamma\kappa}{\sqrt{2}}\left(\frac{\partial u}{\partial y}\right)^3\right)+\\ \frac{16\zeta^\star T_\infty^3}{3a^\star}\frac{\partial^2 T}{\partial y^2}+\frac{\rho_p}{\tau_T}\left(u_p-u\right)^2+\frac{\rho_p c_p}{\tau_T}\left(T_p-T\right)+\sigma B_0^2u^2+q''',\end{aligned} \tag{3}$$

Governing Equations Cont'd

For dust phase:

$$\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} = 0, \tag{4}$$

$$u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial v_p}{\partial y} = \frac{K}{m}\left(u - u_p\right), \tag{5}$$

$$u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} = -\frac{c_p}{c_m \tau_T}\left(T_p - T\right), \tag{6}$$

The wall constraints are:

$$\begin{aligned} &u = u_w + S, v = 0, -k\frac{\partial T}{\partial y} = h_f\left(T_w - T\right) \quad when\ y = 0, \\ &u \to 0, u_p \to 0, v_p \to 0, T \to T_\infty, T_p \to T_\infty, \quad \text{with} \quad y \to \infty. \end{aligned} \tag{7}$$

Transformation & Dimensional Variables

$$\eta = \sqrt{\frac{b}{\vartheta}}y,\ \psi = \sqrt{\vartheta b}xf(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \theta_p(\eta) = \frac{T_p - T_\infty}{T_{pw} - T_\infty},$$

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}, Da = \frac{\nu}{bK_p}, Pr = \frac{\mu c_p}{k_\infty}, M = \frac{\sigma B_0^2}{b\rho}, We = \Gamma\sqrt{\frac{2b^3x^3}{\vartheta}}, \tag{8}$$

$$Ec = \frac{u_w^2}{c_p(T_w - T_\infty)}, Re = \frac{u_w x}{\vartheta}, \gamma = \frac{c_m}{c_p}, Nr = \frac{16\zeta^\star T_\infty^3}{3a^\star k_\infty}, \psi_p = \sqrt{\vartheta b}xf(\eta).$$

The Transformed Equations

$$\left((1-\kappa)+\kappa We f''\right) f''' + f f'' - f'^2 - (M+Da) f' + lB\left(F'-f'\right) = 0, \tag{9}$$

$$FF'' - F'^2 + B\left(f'-F'\right) = 0, \tag{10}$$

$$\begin{aligned}&(1+\delta\theta+Nr)\,\theta'' + Prf\theta' + PrEcMf'^2 + PrEcf''^2\left((1-\kappa)+\frac{We}{2}f''\right) + \delta\theta'^2 + \\ &Prl_t(\phi-\theta) + PrlB\left(F'-f'\right)^2 + Qe^{-\alpha\eta} = 0,\end{aligned} \tag{11}$$

$$F\phi' + \gamma B_t(\phi-\theta) = 0, \tag{12}$$

The boundary situations transform to:

$$\begin{aligned}&f'(0) = 1+\epsilon f'', f(0) = 0, (1+\delta\theta)\,\theta'(0) = -\zeta\left(1-\theta(0)\right),\\ &f' \to 0, F' \to 0, F \to f, \theta \to 0, \theta_p \to 0 \; as \; \eta \to \infty.\end{aligned} \tag{13}$$

Method of Solution

- Due to the high nonlinearity of the problem.
- The solution to the equations (9-13) is found by shooting method with the Runge-Kutta Felherb algorithm.
- Accuracy, reliability and ability of RKF to nonlinear problems is high.
- The codes were written and implemented using the symbolic software Maple 18.
- The adopted values of parameters are :$We = 0.5, B = 0.2,\ Da = 0.4, M = 0.5, L = 0.4, R = 0.2, \delta = 0.2,\ \epsilon = 0.2, Bt = 0.5, Q = 0.1, Ec = 0.2, \kappa = 0.2.$

Results Validation

Table 1. Computed values of Nusselt Number for Pr as compared with published articles when Nr= $We = Q = M = L = s = B = 0, k = 1$

Pr	Mamantha et al. (2018)	Lakshmi et al. (2016)	Ishak (2008)	Current work
0.72	0.8075	0.808630	0.8086	0.808576
1.0	1.0000	1.000000	1.0000	1.000000
3.0	1.9136	1.923670	1.92368	1.923723
10.0	3.6516	3.720670	3.72070	3.723786
100	12.2936	12.29408	12.2941	12.29673

Numerical data for the local skin friction & Nusselt number

M	B	We	Ec	Q	$f''(0)$	$-\theta'(0)$
0 1.0 1.5	0.5	0.5	0.3	0.2	0.920824 1.171450 1.378243	0.130924 0.085195 0.011372
	0.1 0.5 1.0				1.153648 1.108746 1.069771	0.031814 0.099448 0.127399
		0.1 0.5 1.0			1.069721 1.098888 1.176733	0.114915 0.117895 0.057557
			0.0 0.2 0.4		1.069771 1.108746 1.108746	0.155684 0.117770 0.081291
				0.7	1.069771	0.136962

K

Results and Discussion

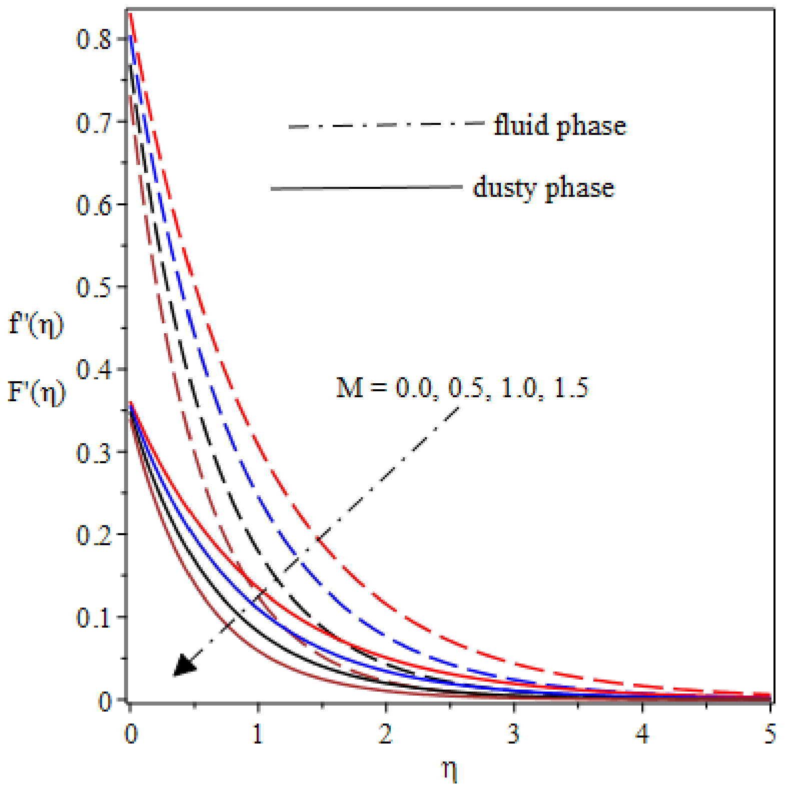


Fig.2. Velocity profiles for variation in **M**

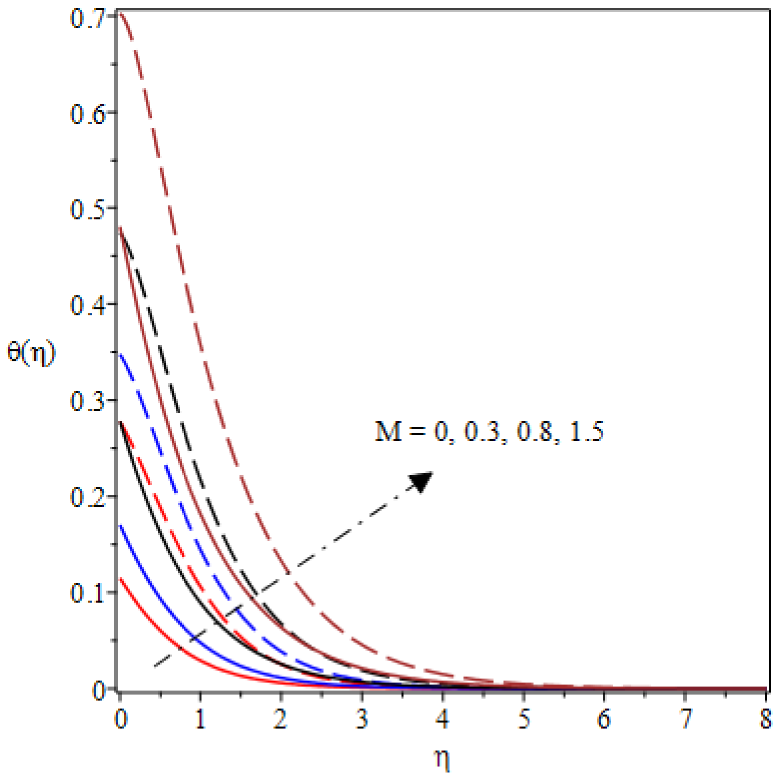


Fig 3. Temperature profiles for M

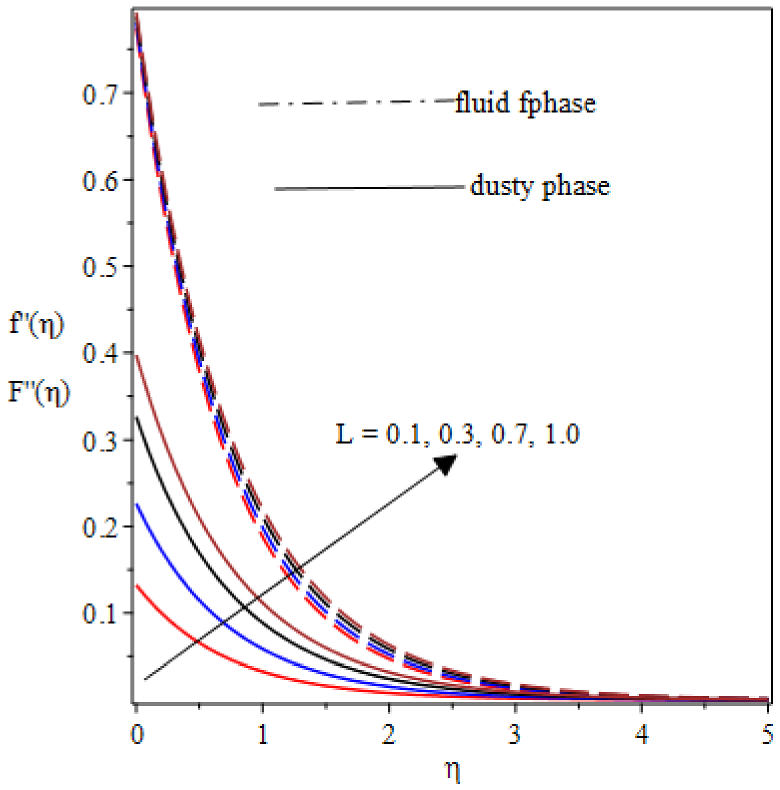


Fig. 4. Velocity profiles for L

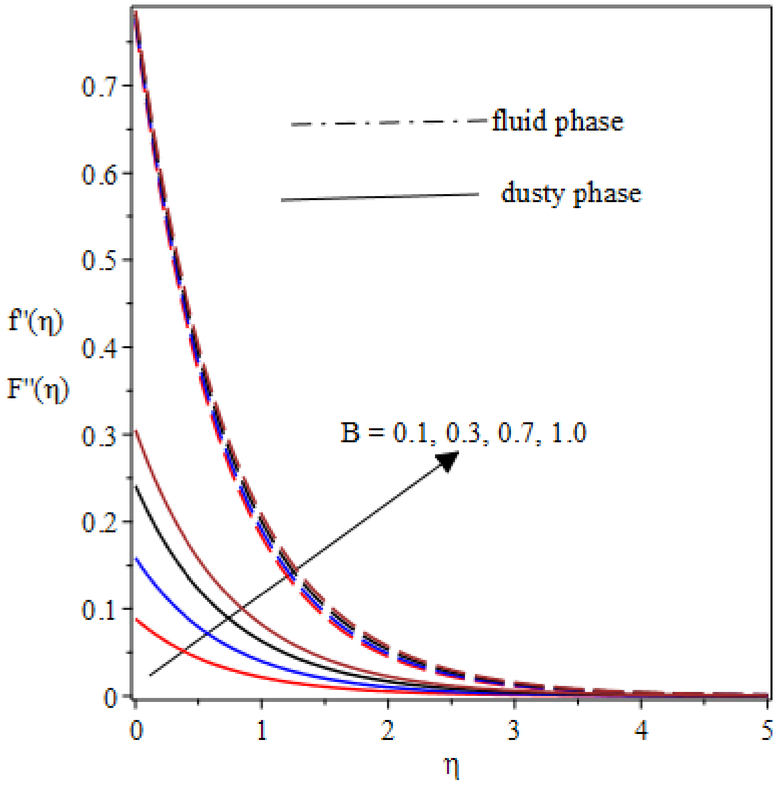


Fig. 5. Velocity profiles for B

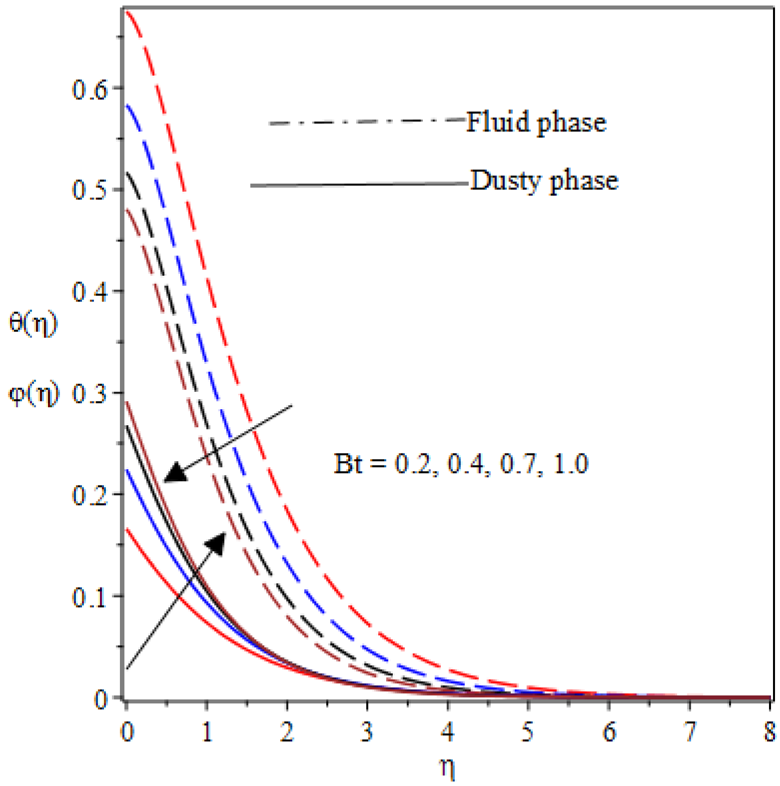


Fig. 6. Effect of Bt on temperature

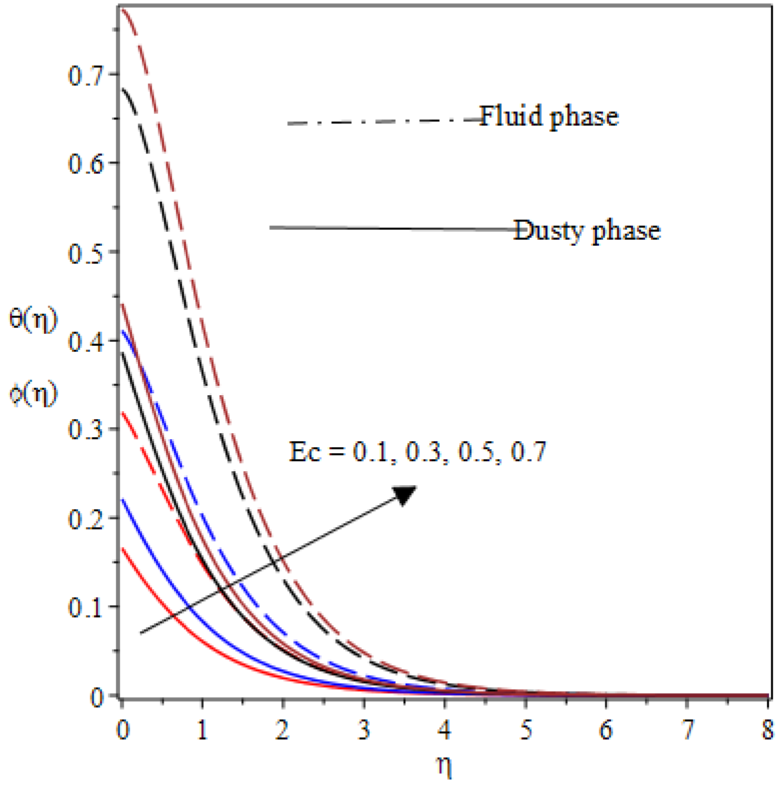


Fig. 7. Effect of Ec on Temperature

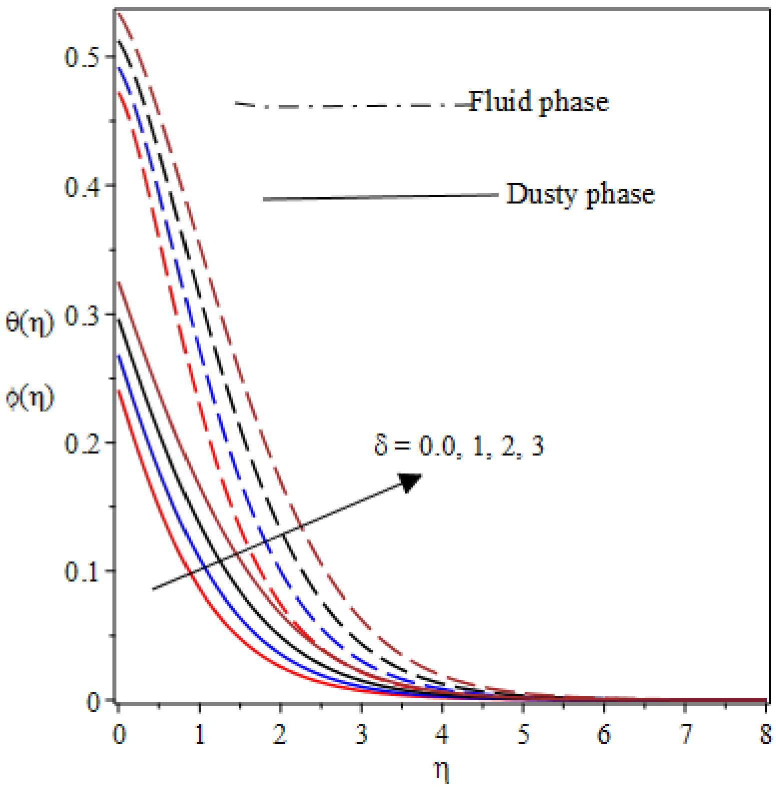


Fig. 8. (a) Effect of δ on thermal profile

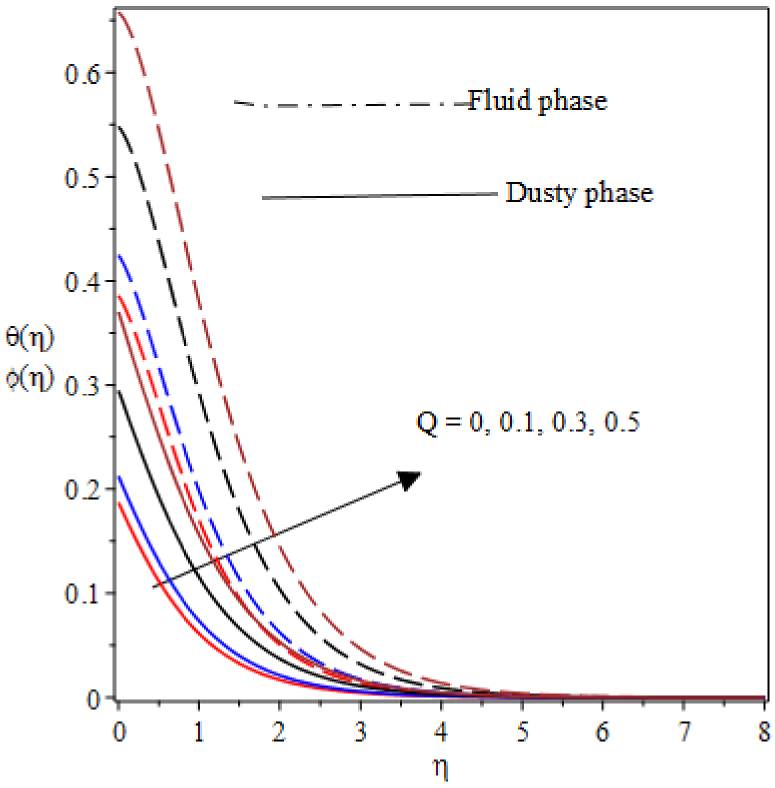


Fig. 9 Effect of Q on heat profiles

Conclusion

- Higher values of the heat source term, magnetic field and thermal conductivity magnify the thermal bounding structure.
- The dusty phase thermal profiles increase with a rise in B_t but the fluid phase heat profile falls
- The hydrodynamic and velocity profiles fall with growing values of the magnetic field and porosity parameters escalate.
- The skin friction coefficient escalate with M, We and B.
- The heat transfer reduces with higher values of Ec and Q.

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