Introduction to complex fluids

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International Conference and Advanced Workshop on Modelling and Simulation of Complex Systems,

May 2024, Obafemi Awolowo University Ile-Ife, Nigeria

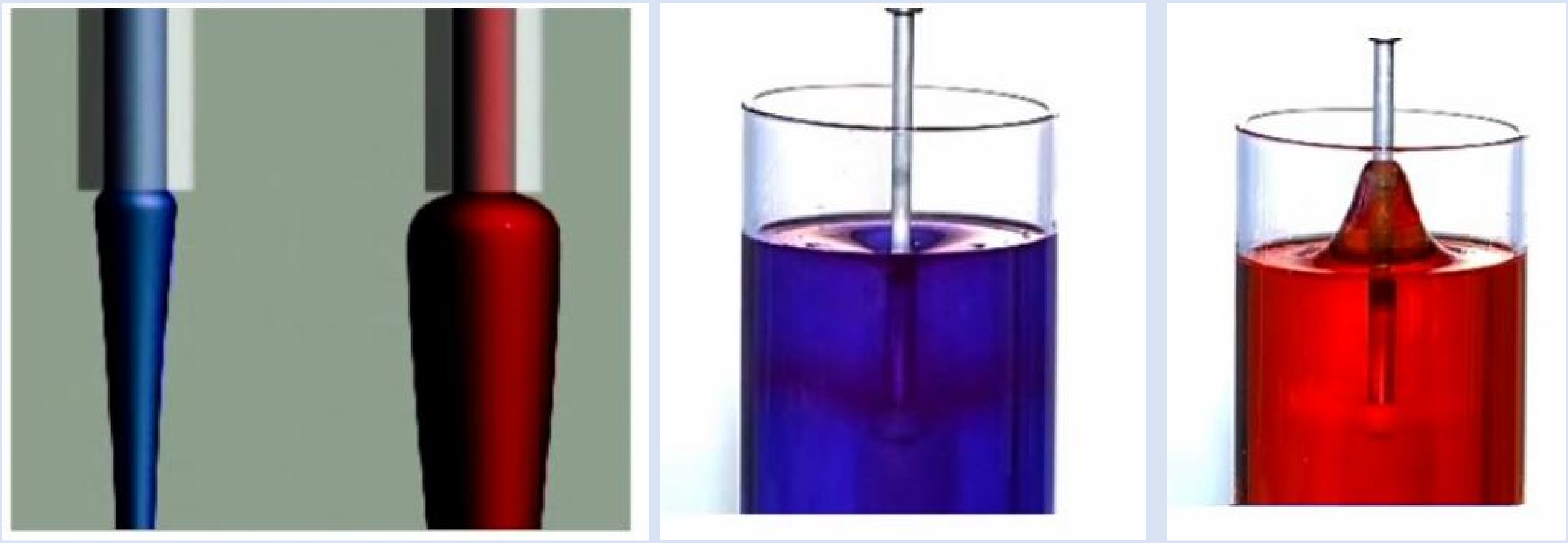
Overview

- Newtonian vs Non-Newtonian (complex) fluids
  - Complex rheological models
    - Generalized Newtonian fluids
    - Viscoelastic fluids
    - Chemorheological fluids (Polyurethane foams)
- The Navier Stokes Equation
- The Navier-Stokes Brinkman Equations

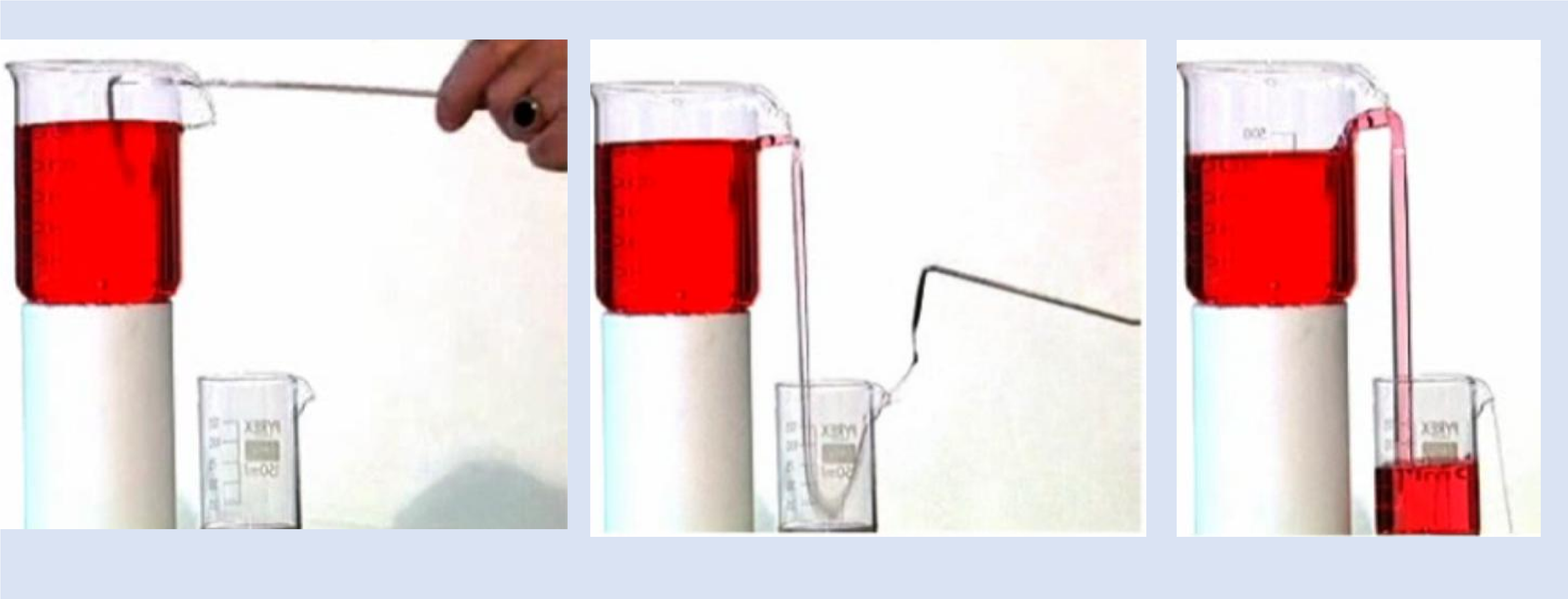
Simple Fluids

https://www.nicepng.com/ourpic/u2q8i1t4u2w7r5y3_water-pouring-png-cup-of-water-png/

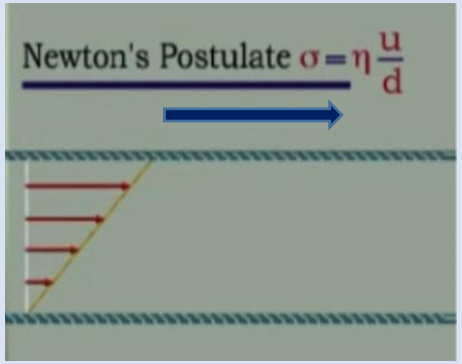
Simple vs Complex Fluids



Complex Fluids



Newtonian Fluids



σ is the shear stress

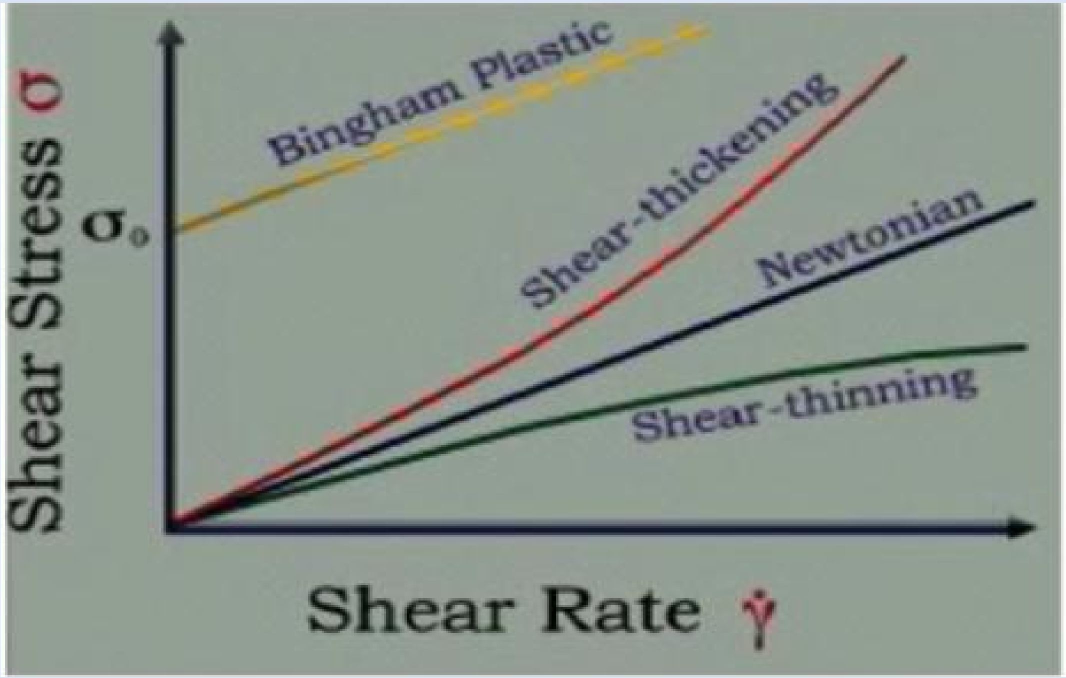
u/d is the velocity gradient

η is the coefficient of viscosity

Table 1.1: Temperature dependent viscosity models

Reynold's model	$\eta(T) = \eta_0 \exp(-BT)$
Vogel's model	$\eta(T) = \eta_0 \exp\left(\frac{B}{T-T_0}\right)$
Arrhenius model	$\eta(T) = \eta_0 \exp(E/RT)$
Nahme model	$\eta(T) = \eta_0 \exp\left(\frac{T-T_0}{T_0}\right)$
William-Landel-Ferry (WLF) model	$\eta(T) = \eta_0 \exp\left(\frac{-C_1(T-T_0)}{C_2+T-T_0}\right)$
Fulcher model	$log_{10}(\eta(T)) = -A + \frac{B\times10^3}{T-T_0}$

Stress strain Curves: Simple vs Complex Fluids



- Viscoelasticity
- Variable Viscosity
- Normal Stresses
- Extensional Viscosity
- Thixotropy and Anti-Thixotropy
- Other Effects

The Navier-Stokes Equations

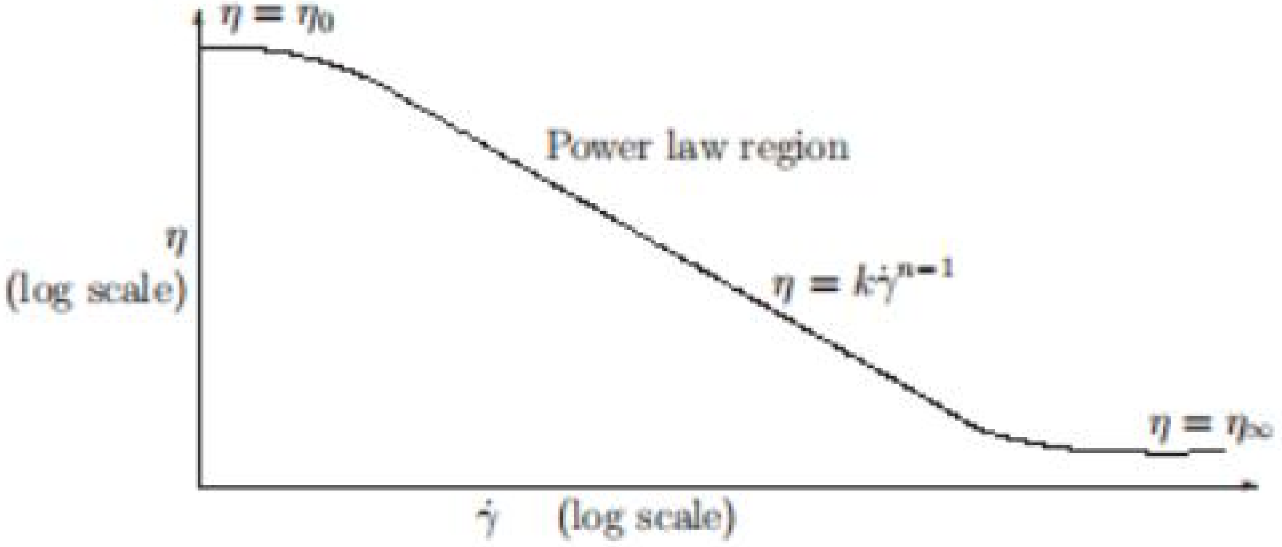
The **Navier-Stokes Equations** subject to appropriate boundary conditions

- **Conservation of mass** $\nabla \cdot \vec{v} = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial t} + \vec{v} \cdot \nabla \rho\right)$
- **Conservation of linear momentum** $\frac{\partial \rho \vec{v}}{\partial t} + \nabla \cdot (\rho \vec{v}\vec{v}) = -\nabla p + \nabla \cdot \boldsymbol{T} + \boldsymbol{F}$
- **Conservation of energy:** $\rho C_p \left(\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T\right) = \nabla \cdot (\boldsymbol{\kappa} \nabla T) + \frac{1}{2}(\mu_{mix} \boldsymbol{D} : \boldsymbol{D})$

Constitutive relations

- Newtonian $$\boldsymbol{T} = 2\eta \boldsymbol{D}, \quad \text{with,} \quad \boldsymbol{D} = \frac{1}{2}\left[\nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^T\right].$$
- **Generalized Newtonian** $$\boldsymbol{T} = 2\eta(\dot{\gamma})\,\boldsymbol{D}.$$
  - Power law Model $$\eta = K\dot{\gamma}^{n-1},$$
    if $n < 1$ $\quad \lim_{\dot{\gamma}\to 0} \eta(\dot{\gamma}) = \infty \quad \lim_{\dot{\gamma}\to\infty} \eta(\dot{\gamma}) = 0,$
    if $n > 1$ $\quad \lim_{\dot{\gamma}\to 0} \eta(\dot{\gamma}) = 0 \quad \lim_{\dot{\gamma}\to\infty} \eta(\dot{\gamma}) = \infty.$
  - Prandtl-Eyring $$\eta = \eta_0 \frac{\sinh^{-1}(\lambda\dot{\gamma})}{\lambda\dot{\gamma}}, \quad \lim_{\dot{\gamma}\to 0} \eta(\dot{\gamma}) = \eta_0, \quad \lim_{\dot{\gamma}\to\infty} \eta(\dot{\gamma}) = 0.$$

Viscosity shear rate relationship: Shear thinning fluid



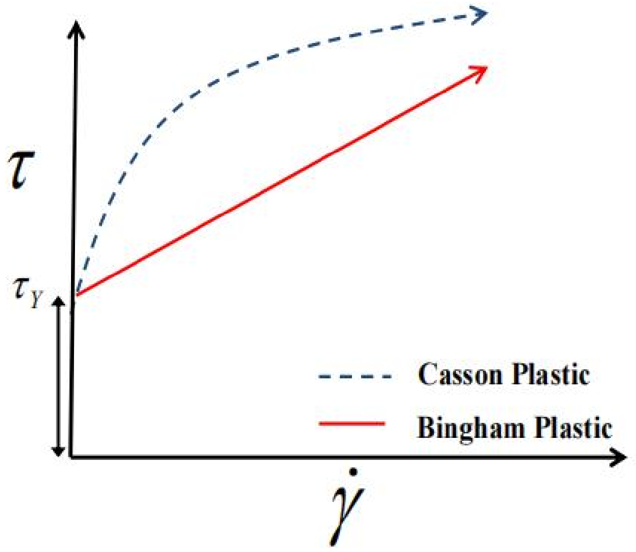
More generalized Newtonian fluids

- Powell-Eyring model $\eta = \eta_\infty + (\eta_0 - \eta_\infty)\frac{\sinh^{-1}(\lambda\dot{\gamma})}{\lambda\dot{\gamma}}.$ $\lim_{\dot{\gamma}\to 0} \eta = \eta_0$ and $\lim_{\dot{\gamma}\to\infty} \eta = \eta_\infty.$
- Cross model $\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \frac{1}{1+(\lambda\dot{\gamma})^m},$ $\frac{\eta_0 - \eta}{\eta - \eta_\infty} = (\lambda\dot{\gamma})^m.$

  if $m < 1$ $\lim_{\dot{\gamma}\to 0} \eta(\dot{\gamma}) = \eta_0$ $\lim_{\dot{\gamma}\to\infty} \eta(\dot{\gamma}) = \eta_\infty,$

  if $m > 1$ $\lim_{\dot{\gamma}\to 0} \eta(\dot{\gamma}) = \eta_\infty$ $\lim_{\dot{\gamma}\to\infty} \eta(\dot{\gamma}) = \eta_0,$
- Sisko model $\eta = \eta_\infty + K\dot{\gamma}^{n-1}.$
- Carreau-Yasuda model $\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = [1 + (\lambda'\dot{\gamma})^c]^{(n-1)/c}.$ c = 2 (Carreau Model)

Yield Stress Fluids

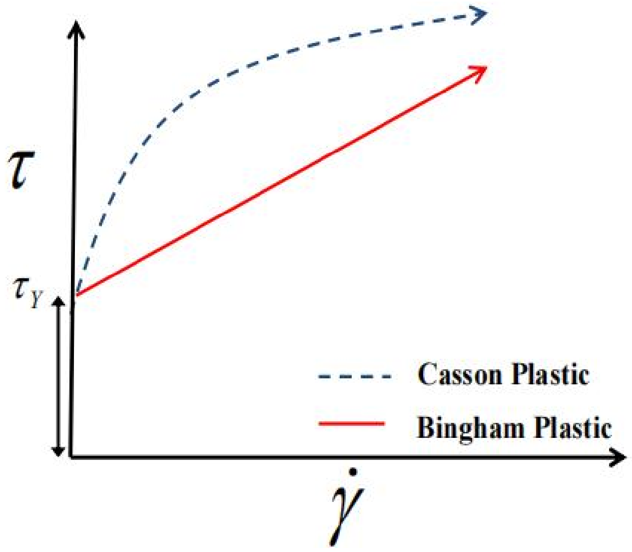


$$\tau = \tau_Y + \eta_p \dot{\gamma}.$$

Bingham plastic:
$$\eta = \begin{cases} \infty & \text{if} \quad \tau \leq \tau_Y, \\ \eta_0 + \frac{\tau_Y}{\dot{\gamma}} & \text{if} \quad \tau > \tau_Y. \end{cases}$$

Herschel-Bulkley model:
$$\eta = \begin{cases} \infty & \text{if} \quad \tau \leq \tau_Y, \\ K\dot{\gamma}^{n-1} + \frac{\tau_Y}{\dot{\gamma}} & \text{if} \quad \tau > \tau_Y. \end{cases}$$

Yield Stress Fluids



Casson plastic:

$$\sqrt{\eta} = \begin{cases} \infty & \text{if} \quad \tau \leq \tau_Y, \\ \sqrt{\eta_0} + \sqrt{\frac{\tau_Y}{\dot{\gamma}}} & \text{if} \quad \tau > \tau_Y. \end{cases}$$

Viscoelastic fluids

Jeffery model
Maxwell Fluid
Order grade fluid
Differential Viscoelastic Model

$$\sigma_{12} = \dot{\gamma}\eta(\dot{\gamma}), \qquad \text{Shear Stress,}$$

$$\sigma_{11} - \sigma_{22} = N_1(\dot{\gamma}), \qquad \text{First Normal stress difference,}$$

$$\sigma_{22} - \sigma_{33} = N_2(\dot{\gamma}), \qquad \text{Second Normal stress difference.}$$

Table 1.2: Some differential viscoelastic models

Model Name	Constitutive model for $\boldsymbol{\sigma}$	N_1 and N_2
Oldroyd-B	$\boldsymbol{\sigma} + \lambda \overset{\nabla}{\boldsymbol{\sigma}} = 2\eta_v \boldsymbol{D}$	$N_1 > 0$ $N_2 \equiv 0$
Geisekus	$\boldsymbol{\sigma} + \alpha (\boldsymbol{\sigma} \cdot \boldsymbol{\sigma}) + \lambda \overset{\nabla}{\boldsymbol{\sigma}} = 2\eta_v \boldsymbol{D}$	$N_1 > 0$ $N_2 \neq 0$
Johnson-Segalman	$\boldsymbol{\sigma} + \lambda \overset{\square}{\boldsymbol{\sigma}} = 2\eta_v \boldsymbol{D}$	$N_1 > 0$ $N_2 \neq 0$
Diffusive Johnson-Segalman	$\boldsymbol{\sigma} + \lambda \overset{\square}{\boldsymbol{\sigma}} = 2\eta_v \boldsymbol{D} + \varepsilon \nabla^2 \boldsymbol{\sigma}$	$N_1 > 0$ $N_2 \neq 0$
	$\boldsymbol{\sigma} + \lambda \overset{\square}{\boldsymbol{\sigma}} = 2\eta_v \boldsymbol{D} + \varepsilon \nabla^2 \boldsymbol{D}$	$N_1 > 0$ $N_2 \neq 0$
Phan-Thien-Tannar	$\exp\left(\pi \frac{\lambda}{\eta_v} \mathrm{tr}(\boldsymbol{\sigma})\right) + \boldsymbol{\sigma} + \lambda \overset{\square}{\boldsymbol{\sigma}} = 2\eta_v \boldsymbol{D}$	$N_1 > 0$ $N_2 \neq 0$

Chemorheological fluids

$$\eta = \eta(\theta, p, \dot{\gamma}, t, F, \alpha)$$

$$\boldsymbol{T} = \eta_m \mathbf{D}$$

$$\eta_m = \begin{cases} \eta_F, & \text{for} \quad \varphi \geq \varphi_s, \\ \eta_F \frac{\varphi}{\varphi_s} + \eta_g \left(1 - \frac{\varphi}{\varphi_s}\right), & \text{for} \quad 0 \leq \varphi < \varphi_s. \end{cases}$$

$$\eta_F = \eta_{oo} \exp\left(\frac{E_\eta}{RT}\right) \cdot \left(\frac{\zeta_c}{\zeta_c - \zeta}\right)^{h(\zeta)} f(\varphi_g),$$

Thank You
for Your Attention