RECYCLING OF MUNICIPAL SOLID WASTE: A DETERMINISTICAPPROACH.

By

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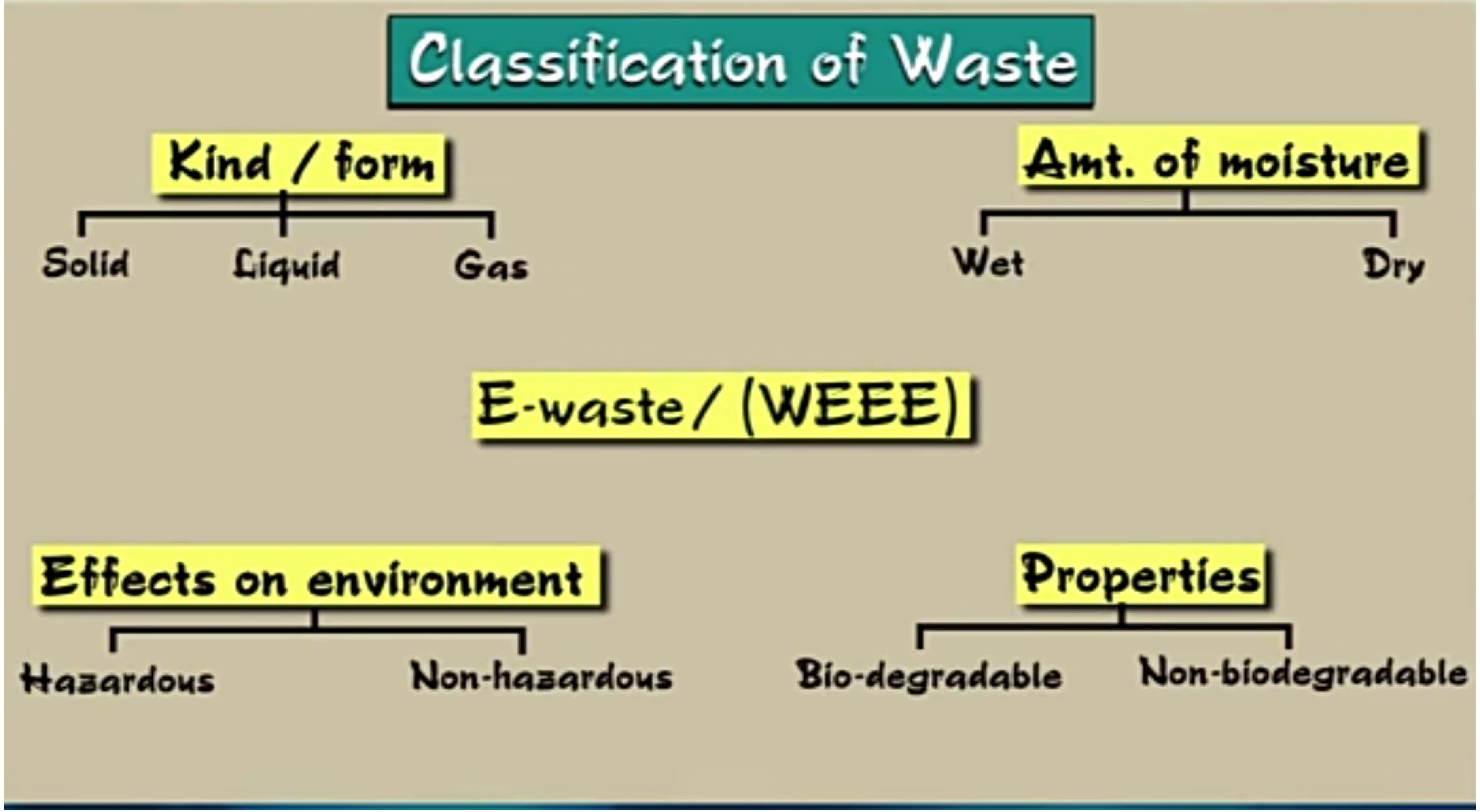
ABSTRACTS

- Effective waste management aims at minimizing garbage's detrimental effects on the environment, public health, and aesthetics, which also attempts to recover valuable resources and support sustainable development.
- The New Generation matrix was employed to calculate the reproduction number, the model equations were solved using the Differential Transformation Method (D.T.M.) and the obtained result was simulated using the Maple software. The result shows that waste management will be effective if recycling waste is given the proper attention that it deserves. It also indicates that waste for disposal will be limited and managing waste will become easier.

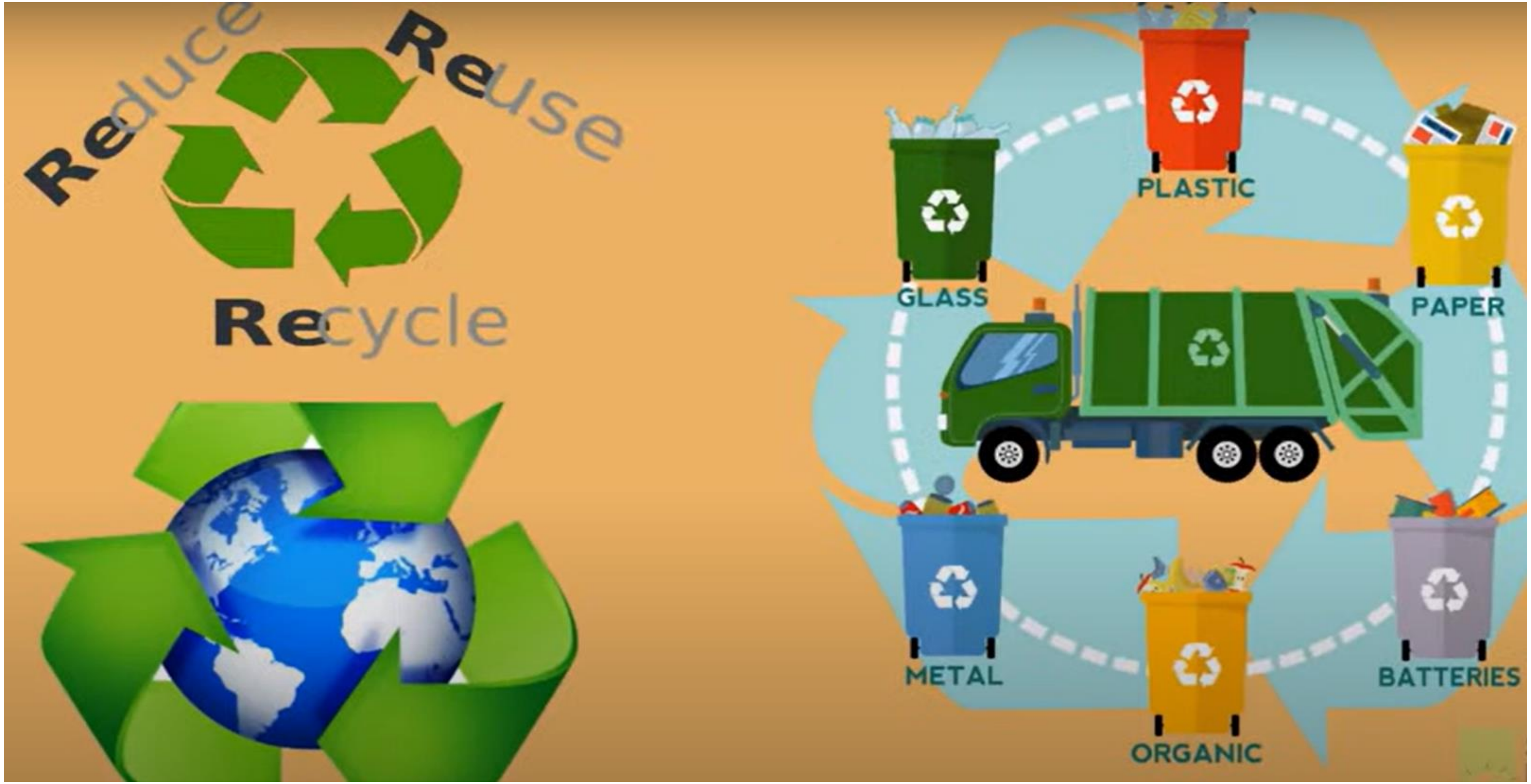
INTRODUCTION



Why do we need to recycle

- Saves the Planet
- Protects People
- Saves Resources
- Boosts Sustainability
- Recycling is the Key

THE 3 R'S OF RECYCLING



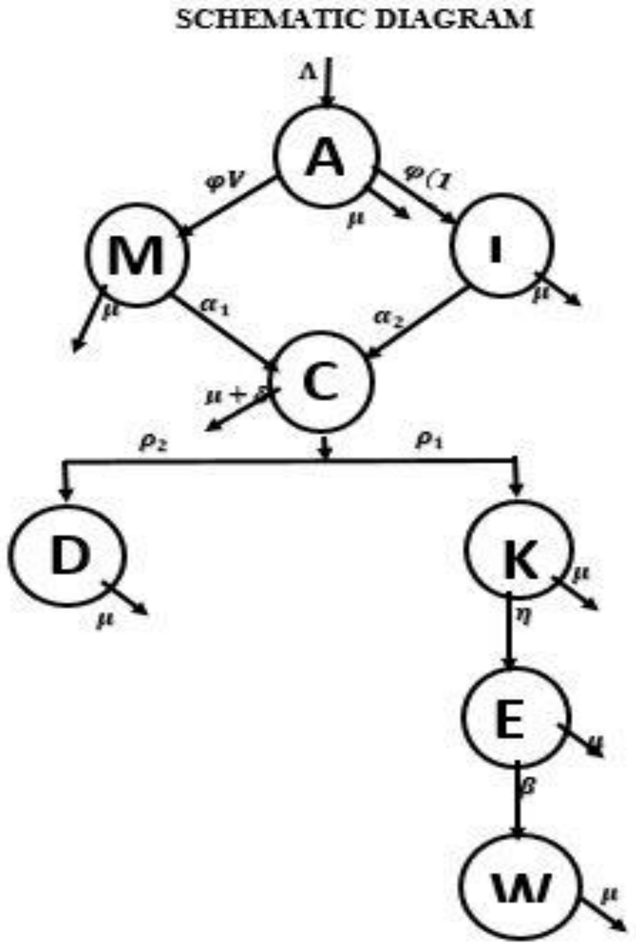
WHAT OTHERS HAVE DONE ON WASTE

Author/year	Research Done	Method	Result
Jonas Petro Senzige, Daniel Oluwole Makinde, Karoli Nicolas Njau, Yaw Nkansah-Gyeke 2014	identified and elucidated the factors influencing solid waste generation and composition in the three municipalities of Dar es Salam and compare the results with previous research and data from other East African countries.	1) classifying the population by socioeconomic status; 2) selecting households for the study; 3) determining the number of samples; 4) sorting and quantifying the solid waste types and 5) analysing the results	indicated that solid waste generation and composition is highly dependent on population and socioeconomic status of the population
Jonas Petro Senzige and Oluwole Daniel Makinde 2016	They formulated a mathematical model of the effects of population dynamics on solid waste generation and treatment considering three groups of young, Adults and the Elderly.	The model considered an increase in population due to birth and migration. It also considered that each of the three groups has had different waste generation and death rates. Equilibrium points were obtained, stability analysis of Waste-free and Waste-Endemic equilibrium were discussed, parameters were further estimated and a conclusion was drawn from the results.	1) solid waste generation for each group will initially increase but as each group comes to a steady state the solid generation rate becomes constant 2) it is important for the authorities to encourage community initiatives focusing on, compositing, recycling, reuse and waste-to-energy conversion so that manageable solid waste quantity remains for disposal.

WHAT OTHERS HAVE DONE ON WASTE

Momoh 2019 titled "Mathematical Modelling of Waste Management: A Deterministic Approach"	She Formulated a deterministic model with six compartments for waste management to reduce and eliminate the consequences of poor waste management	Validation of the model equations was performed, Equilibrium points were obtained, stability analysis of Waste-free and Waste-persistent equilibrium was discussed, and analytical simulations were performed.	Waste reduction must be from the source. To control/reduce wastage consumption must be reduced to the essentials.
Akinboro, Alao, & Akinpelu, 2014	Presented two types of S-I-R model to compare DTM and VIM	Solution of the non-linear S-I-R model were solved using DTM and VIM	It was recommended that the two method can be used to solve the non-linear differential equations effectively.

SCHEMATIC DIAGRAM



MODEL EQUATIONS

$$
\left.\begin{aligned}
\frac{dA}{dt} &= \Lambda - \xi(M + \omega I)A - \mu A \\
\frac{dM}{dt} &= \xi\nu(M + \omega I)A - (\alpha_1 + \mu)M \\
\frac{dI}{dt} &= \xi(1-\nu)(M + \omega I)A - (\alpha_2 + \mu)I \\
\frac{dC}{dt} &= \alpha_1 M + \alpha_2 I - (\rho_1 + \rho_2 + \mu + \delta)C \\
\frac{dK}{dt} &= \rho_1 C - (\eta + \mu + \delta)K \\
\frac{dE}{dt} &= \eta K - (\beta + \mu)E \\
\frac{dW}{dt} &= \beta E - \mu W \\
\frac{dD}{dt} &= \rho_2 C - \mu D
\end{aligned}\right\} \qquad 1
$$

MODEL ASSUMPTIONS

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1. The model assumed that manufactured goods are used for domestic and industrial purposes only.
2. The model considers solid waste only.
3. The wastes generated in the industries are controlled and minimized.
4. The wastes from the collation centre are either recycled or disposed of.
5. It is assumed that wastes are properly disposed

Existence of Waste-Free Equilibrium

At the Waste-Free Equilibrium (W.F.E.) point

$$\frac{dA}{dt}=\frac{dM}{dt}=\frac{dI}{dt}=\frac{dC}{dt}=\frac{dK}{dt}=\frac{dD}{dt}=\frac{dE}{dt}=\frac{dW}{dt}=0 \tag{2}$$

$$A,M,I,C,D,K,E,W=A_0,M_0,I_0,C_0,D_0,K_0,E_0,W_0 \tag{3}$$

$$(A_0,M_0,I_0,C_0,D_0,K_0,E_0,W_0)=\left(\frac{\Lambda}{\mu},0,0,0,0,0,0,0\right) \tag{4}$$

$$\begin{aligned}
&\Lambda-\xi(M_0+\omega I_0)A_0-\mu A_0=0\\
&v\xi(M_0+\omega I_0)A_0-(\alpha_1+\mu)M_0=0\\
&(1\text{-}v)\xi(M_0+\omega I_0)A_0-(\alpha_2+\mu)I_0=0\\
&\alpha_1M_0+\alpha_2I_0-(\rho_1+\rho_2+\mu+\delta)C_0=0\\
&\rho_1C_0-(\omega+\mu+\delta)K_0=0\\
&\omega K_0-(\beta+\mu)E_0=0\\
&\beta E_0-\mu W_0=0\\
&\rho_2C_0-\mu D_0=0
\end{aligned} \tag{5}$$

$$k_1=(\alpha_1+\mu),k_2=(\alpha_2+\mu),k_3=(\rho_1+\rho_2+\mu+\delta),k_4=(\omega+\mu+\delta),k_5=(\beta+\mu), \tag{6}$$

WASTE REPRODUCTION NUMBER

When applied to solid waste management the Waste Reproduction Number refers to the rate at which solid waste spreads in the environment. Ibrahim & Abdurrahman (2023). The value of R_0 determines whether the spread of garbage dies out in the community or spreads so widely that it can be considered an epidemic. Whenever the value of the R_0 goes below unity $R_0 < 1$, the spread of the waste is insignificant and waste management becomes easier. When the value is above unity $R_0 > 1$ it means that the spread of waste in the community becomes significant.

$$R_0 = \frac{\Lambda\xi\left(\mu\omega v + \omega v\alpha_1 - \mu\omega - v\mu - \omega\alpha_1 - v\alpha_2\right)}{\mu\left(\alpha_2 + \mu\right)\left(\alpha_1 + \mu\right)} \qquad 7$$

DIFFERENTIAL TRANSFORMATION METHOD (D.T.M.)

$$H(k)=\sum_{k=0}^{\infty}\frac{1}{k!}\left[\frac{d^k h(t)}{dt^k}\right)_{t=0}$$

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where H(k) is the transformed function, also called the T-function, and k is a non-negative integer. The inverse differential transform of H(k) is given by

$$h(t)=\sum_{k=0}^{\infty}H(k)t^k$$

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where h(t) is the original function. The equation (13) above was derived from Taylor series expansion which can be written in the form:

$$h(t)=\sum_{k=m+1}^{\infty}H(k)(t-t_0)^k$$

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- **THE FUNDAMENTAL PROPERTIES OF DIFFERENTIAL TRANSFORM METHOD (D.T.M.)**

S/NO	Existing Functions	Transformed Functions
1	$h(t) = f(t) \pm g(t)$	$H(k) = F(k) \pm G(k)$
2	$h(t) = af(t)$	$H(k) = aF(k)$
3	$h(t) = \frac{df(t)}{dt}$	$H(k) = (k+1)F(k+1)$
4	$h(t) = \frac{d^2 f(t)}{dt^2}$	H(k)=(k+1)(k+2)...k(k+n)F(k+n) $H(k) = (k+1)(k+2)F(k+2)$
5	$h(t) = \frac{d^n f(t)}{dt^n}$	H(k)=(k+1)(k+2)...k(k+n)F(k+n)
6	$h(t) = 1$	$H(k) = \delta(k)$
7	$h(t) = t$	$H(k) = \delta(k-l)$
8	$h(t) = t^r$	$H(k) = \delta(k-r); \delta(k-r) = \begin{cases} 1 & k = r \\ 0 & \text{else} \end{cases}$
9	$h(t) = f(t)g(t)$	$H(k) = \sum_{m=0}^{k} F(m)G(k-m)$
10	$h(t) = \frac{f(t)}{g(t)}$	$H(k) = \sum_{m=0}^{k} \frac{F(m)}{G(k-m)}$
11	$h(t) = \exp(rt)$	$H(k) = \frac{r^k}{k!}$

$$A(k+1)=\frac{1}{k+1}\left(\Lambda\delta(k)-\xi\sum_{m=0}^{k}\left(M(m)+\omega I(m)\right)A(k-m)-\mu_1 A(k)\right)$$

$$M(k+1)=\frac{1}{k+1}\xi v\sum_{m=0}^{k}\left(M(m)+\omega I(m)\right)A(k-m)-k_1 M(k)$$

$$I(k+1)=\frac{1}{k+1}\left((1\text{-v})\xi\sum_{m=0}^{k}\left(M(m)+\omega I(m)\right)A(k-m)-k_2 I(k)\right)$$

$$C(k+1)=\frac{1}{k+1}\left(\alpha_1 M(k)+\alpha_2 I(k)-k_3 C(k)\right)$$

$$K(k+1)=\frac{1}{k+1}\left(\rho_1 C(k)-k_4 K(k)\right)$$

$$E(k+1)=\frac{1}{k+1}\left(\eta K(k)-k_5 E(k)\right)$$

$$W(k+1)=\frac{1}{k+1}\left(\beta E(k)-\mu W(k)\right)$$

$$D(k+1)=\frac{1}{k+1}\left(\rho_2 C(k)-\mu D(k)\right)$$

$$A(t) = 1.754847056 \times 10^{14} t^4 - 2.212036168 \times 10^{12} t^3 - 1.937760000 \times 10^6 t^2 - 3.621600000 \times 10^6 t + 12000$$

$$M(t) = -8.774235282 \times 10^{13} t^4 - 5.168124901 \times 10^{10} t^3 + 945199.2000 t^2 + 1.817520000 \times 10^6 t + 6000$$

$$I(t) = -8.774096771 \times 10^{13} t^4 - 5.540368577 \times 10^{10} t^3 + 927030 t^2 + 1.81740 \times 10^6 t + 6000$$

$$C(t) = 1.413010420 \times 10^9 t^4 - 30719.30800 t^3 - 90868.2 t^2 - 3300 t + 21000$$

$$R(t) = 16905.30450 t^3 + 54509.85000 t^2 + 270.00 t + 11000$$

$$E(t) = 235.0658400 t^4 - 283.9260000 t^3 + 3260.400000 t^2 - 7000.00 t + 8000$$

$$W(t) = -75.09424916 t^4 + 1010.442333 t^3 - 3243.500000 t^2 + 6540.00 t + 10000$$

$$\mathrm{D(t)} = 11754.74167 t^3 + 36339.5 t^2 + 340 t + 10000$$

DISCUSSION OF RESULTS

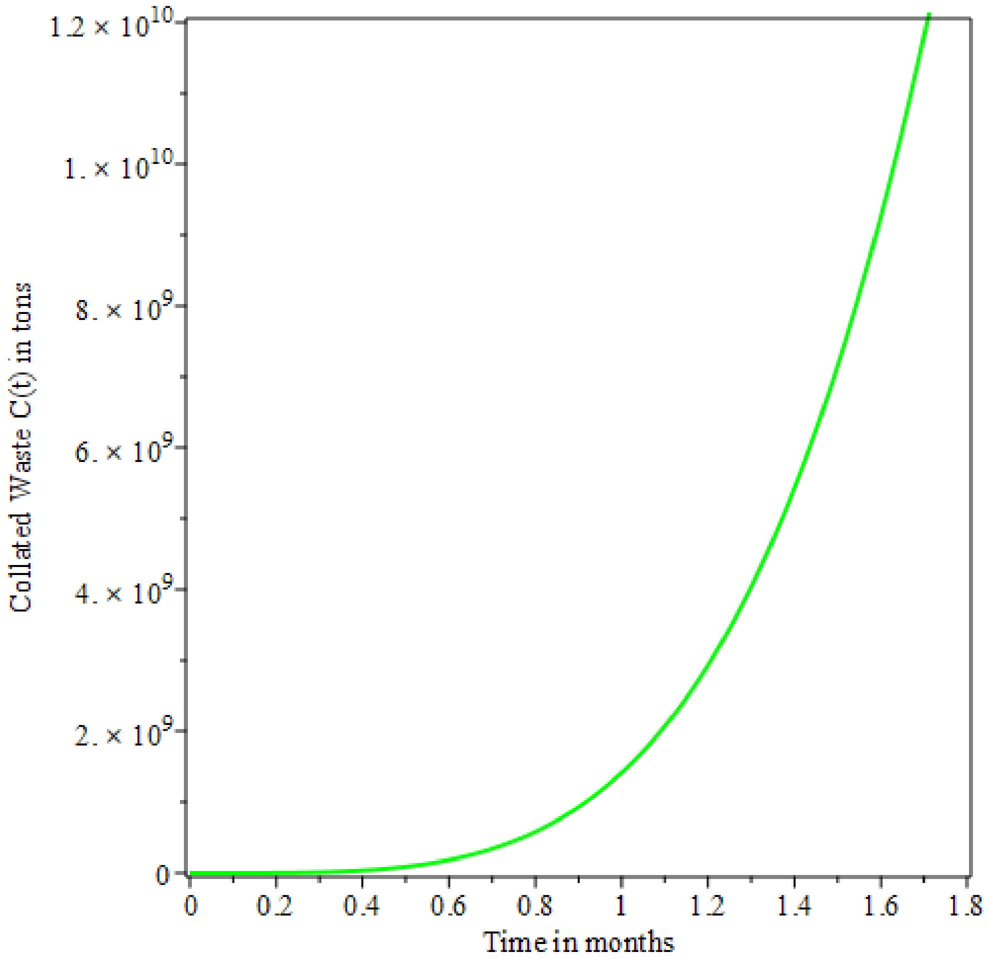

Fig1a:change in Collated wastes with time

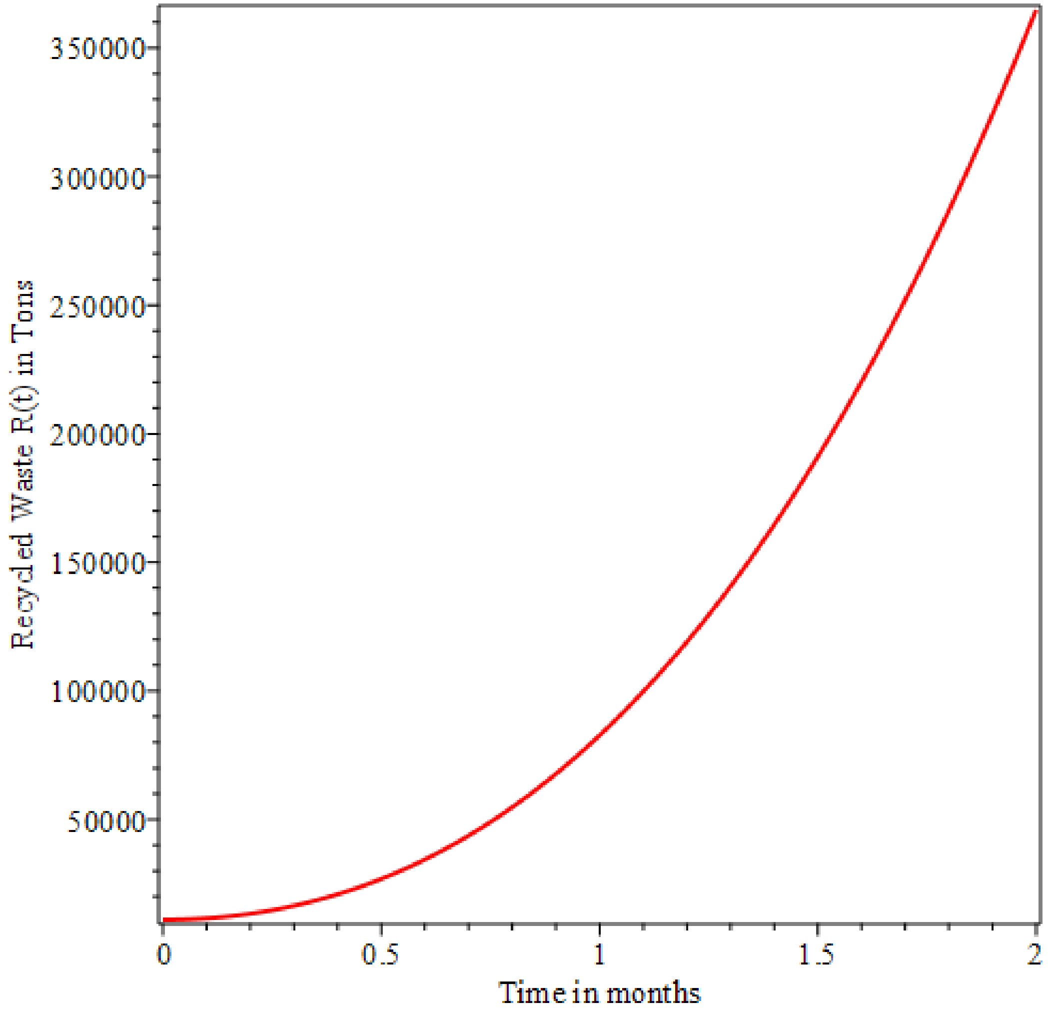

Fig1b:change in Recycled wastes with time

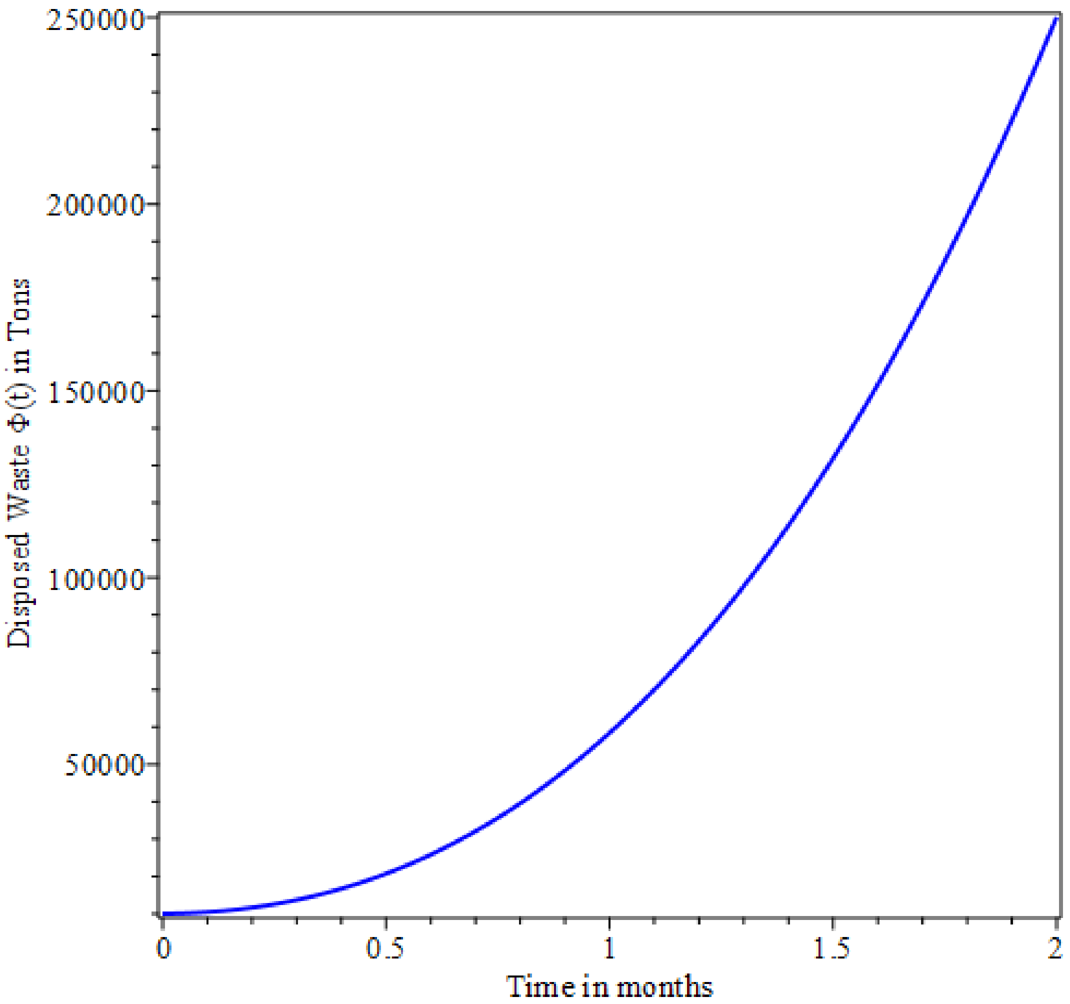

Fig2a:change in Disposed wastes with time

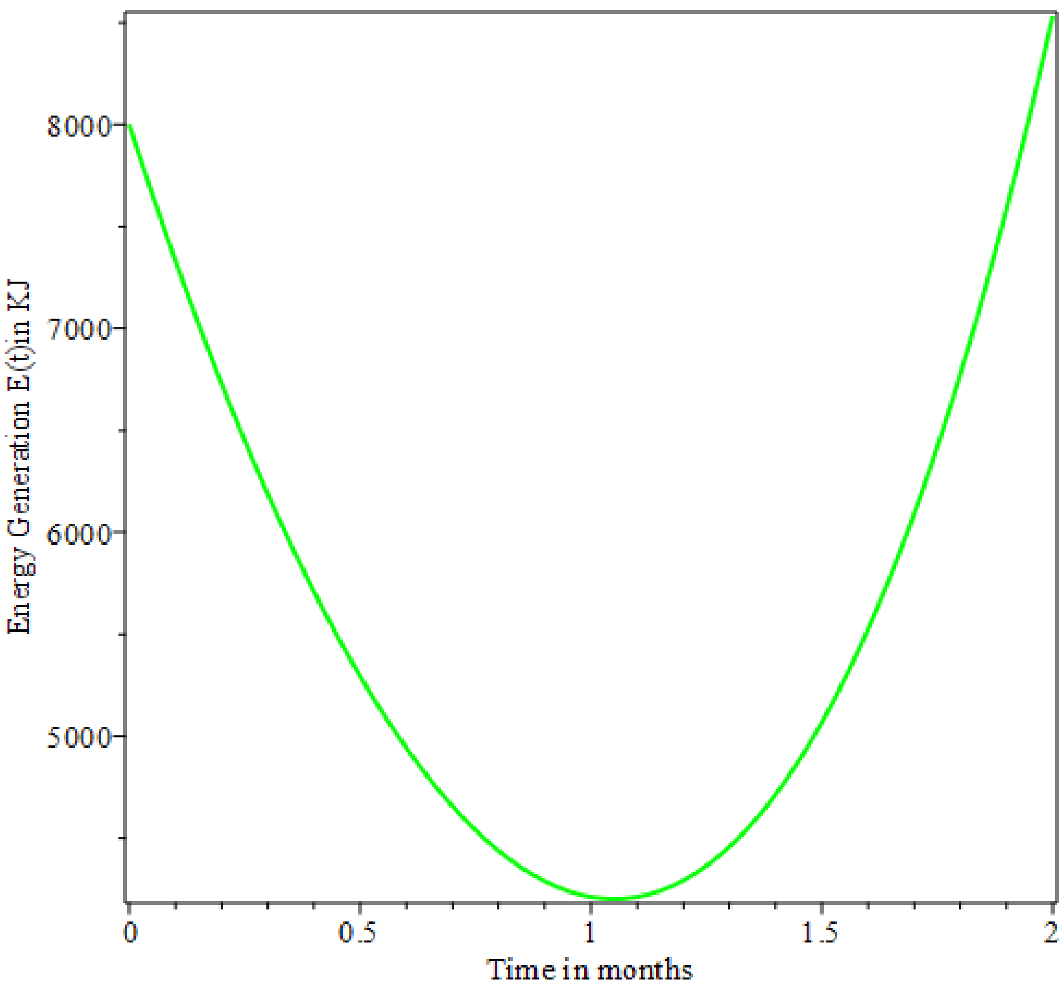

Fig2b:change in Energy Generation with time

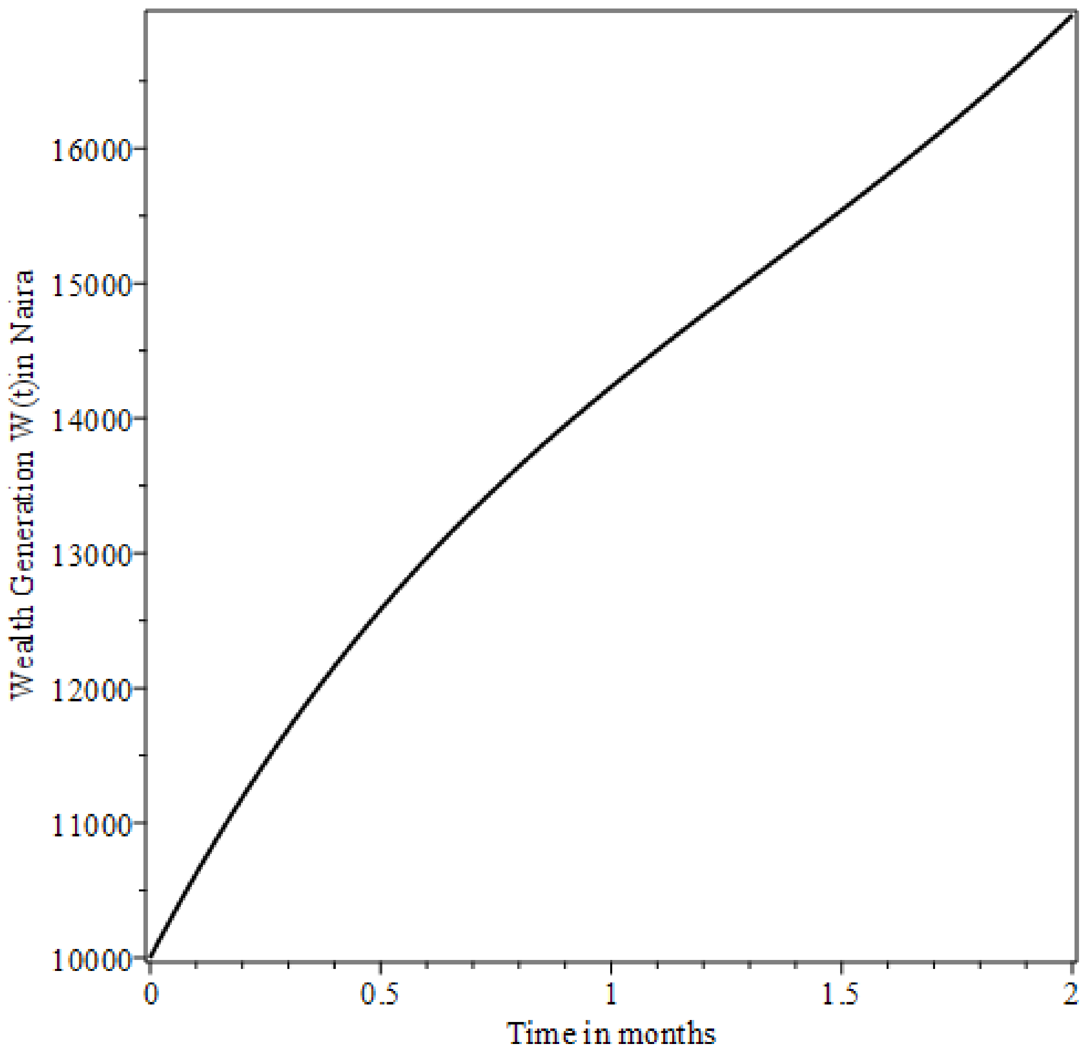

Fig3:change in Wealth Generation with time

RESULTS AND DISCUSSION.

The results obtained from the D.T.M. were simulated using the Maple software. They are shown above in Figures 1 to 3. The fig1a shows that the collated waste increases at an exponential rate with time and it gets to the peak after some time, indicating that some of the wastes decompose at the collation centre and some were lost by other means. The recycled waste increases with time as shown in Figure 1b, this is due to the fact that recycled waste is obtained from the collation centre so it increases at the same rate. Figure 2a shows that disposed waste increases with time, based on the sources of disposal. Figure 2b gives the energy generation as reducing with time till a minimum point is achieved and it then increases till a maximum point is achieved. This implies that energy generation from waste will take sometimes before it can be get the optimal point. Wealth creation as shown in figure 3 increases wi th time as the quantity of waste generated increases.

CONCLUSION

The model of recycling of waste using mathematical approach was formulated, confirmed to be epidemiologically well posed and that its unique solution lies in the positive invariant region. The model equation was solved using the D.T.M. The results from the D.T.M shows that energy generation and wealth creation from waste will take some times before it can be maximized as it relies solely on the quantity of waste recycled for that purpose. This study recommended that concerted effort should be put into waste recycling and energy generation to increase wealth creation and facilitate proper waste management.