UNRAVELLING COMPLEX DYNAMICS AND ACTIVE CONTROL OF A NOVEL MULTI-EQUILIBRIUM HYPERCHAOTIC MEMRISTIVE-BASED VARIABLE-BOOSTABLE SYSTEM

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PRESENTATION OUTLINE

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- STATEMENT OF THE PROBLEM
- AIM AND OBJECTIVES OF THE STUDY
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- RESULTS AND DISCUSSION
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INTRODUCTION

- **Chaos theory** is a branch of mathematics and science that studies complex systems that exhibits dependence on initial conditions.

- Chaos finds use in physics, biology, economics, and weather prediction, shedding light on the intricate, nonlinear dynamics of natural systems.

- Whereas, a variable boostable system in chaos theory refers to a system where parameters can be dynamically adjusted to control or influence chaotic behavior
- Such systems have applications across various fields such as secure commication, control systems, biological system, weather and climate modelling and so on.

INTRODUCTION CONT'D

- However, by incorporating additional terms, adjusting initial conditions, expanding dimensions, or introducing nonlinear pathways (e.g., via memristor inclusion), novel dynamical systems with diverse dynamics can emerge.

- The increasing need for heightened security measures has led to a significant rise in the exploration of multiequilibrium hyperchaotic systems integrated with memristive technology.

- On the other hand, hyperchaotic systems that exhibit a wide range of equilibrium points are termed **"multiequilibrium hyperchaotic systems."**

INTRODUCTION CONT'D

- But, active control strategies are essential for managing the complexity and hyperchaotic nature of the 5D V-B system.

- Moreso, integration of active control methods allows for influencing and regulating system behavior, enhancing stability, thereby mitigating undesired effects.

- Whereas, employing active control strategies to leverage the nonlinearity and memory capabilities of series-based memristors offers a dynamic approach to Shaping the system's behavior.

LITERATURE REVIEW

S/N	AUTHORS	LOCATION	RESEARCH	RESULT	COMMENTS
1	Quintero-Quiroz *et al.*, 2014	Spain	Collective behavior of chaotic oscillators with environmental coupling	The results shows that two collective state emerge from this setup. First, a nontrivial behavior where oscillators exhibit periodic evolution along side local chaos. Secondly, the oscillators exhibit clustering.	The research is limited to numerical analysis. No application to electronics
2	Suresh *et al.*, 2014	India	Dynamic environment coupling induced synchronized states in coupled time-delayed electronic circuits	The results shows that various types of synchronization from inverse phase synchronization to inverse synchronization and from phase synchronization to complete synchronization can be achieved by using dynamical environment coupling.	The research focuses integer order system in which memristor is absent.
3	Liu *et al.*, 2021	China	Double memristor series hyperchaotic system with attractive coexistence and its circuit implementation.	The results shows that the system exhibits hyperchaotic behavior which was analyzed through phase and bifurcation diagrams, Lyapunov exponent spectrum and fractal dimensions. The	Dynamic coupling of the system was not applied

S/N	AUTHORS	LOCATION	RESEARCH	RESULT	COMMENTS
4	Piyush, 2021	India	A Novel Chaotic System with Wide Spectrum, its Synchronization, Circuit Design and Application to Secure Communication	The result shows that synchronization of states between the drive and response system was achieved by using nonlinear active control scheme.	Synchronization was achieved through nonlinear active control scheme.
5	Adelakun, 2021	Nigeria	OPCL coupling of mixed integer-fractional order oscillators: tree and chain implementation	The results shows that a unidirectional Open – Plus – Closed – Loop coupling (OPCL) was used to achieve synchronization between two mixed oscillators.	The research focuses on using OPCL coupling to achieve synchronization.
6	Adelakun *et al.*, 2023	Nigeria	Robust multiple-scroll dynamics in memristive-based generator system	The new proposed model reveals no equilibrium point, sensitive to initial conditions, and the transition from chaos to hyperchaos state coexistence multistability of attractors.	Diiferent coupling used to achieved control.

STATEMENT OF THE PROBLEM

- Globally, the previous research work that has been done in this area of research has been on the existing 3D-variable-boostable system. However, in this research work the theoretical, numerical, and electronic design of a novel 5D-hyperchaotic system will be treated;

- and, further actualize the control of the system by applying active control scheme.

AIM AND OBJECTIVES

AIM

The research aims to generate a new hyperchaotic double-memristive-based system and investigate the environmental coupling of the memristive-based integer order system.

OBJECTIVES

The specific objectives of this work are to:

- investigate the theoretical analysis, numerical simulation and experimental realization of a new integer-order double-memristive-based system;
- achieve stability through the implementation of an active control scheme of the system.
- determine the experimental realization of the 5D-memristive based hyperchaotic system.

RESEARCH METHODOLOGY

Theoretical calculations

- Stability and Equilibrium points analysis
- Lyapunov exponent and Kaplan Yorke dimension

Numerical simulation

- Test for hyperchaotic behaviours using mathematical tools explaining:

  Bifurcation diagrams, Lyapunov exponents, Time series variation, phase plots diagrams, Coexistence of attractors.

Experimental circuit realization and validations

- Hardware implementation for the proposed system
- Arduino link simulations

RESEARCH METHODOLOGY CONT'D

The existing 3D model (Li and Sprott, 2016) is given by :

$$\dot{x} = z^2 - 1$$

$$\dot{y} = az^2 + byz \qquad (1)$$

$$\dot{z} = x - y^2$$

Equation (1) is called 3D-Variable-boostable system with the following system properties:

- Parameters: a = 1, b = 2
- Equilibrium points $E_1 = (0.25, 0.5, -1)$ and $E_2 = (0.25, -0.5, 1)$
- Eigenvalues = (-2.3146, 0.1573$\pm$1.3052i) and (2.5616, 1, -1.5616)
- Initial conditions $(x_0, y_0, z_0) = (1,1,1)$
- Lyaponov Exponents (LEs) = (0.1223, 0, -1.6965)
- Kaplan–Yorke Dimension(Dky) = (2.0721).

The existing 3D system is Chaotic.

RESEARCH METHODOLOGY CONT'D

Dual Series Memristor

Considering the special properties of the dual memristor in Equation (2) and (3). Liu *et al*., *2021*, implement a double series hyperchaotic Rossler system, where the two memristors are of trigonometric sine and cosine function of I-V relationship:

$$\begin{cases} i = W_1(\phi_1)V = -\dfrac{R_{m_2g_1}}{R_{m_1}R_bC}\sin\left(\dfrac{R_{m_5}}{R_{m_4}}\phi_1\right)V \\ = \dfrac{R_{m_2}}{R_{m_1}R_{m_3}C_{m_1}}a\sin\left(\dfrac{R_{m_5}}{R_{m_4}}\phi_1\right)V \\ \phi_1 = \dfrac{R_{m_2}}{R_{m_1}R_{m_3}C_{m_1}}V_1 \end{cases} \qquad (2)$$

$$\begin{cases} i = W_2(\phi_2)W_1 = -\dfrac{g_2}{R_{m_7}C_{m_3}}\cos(\phi_2)V_1 = \dfrac{1}{R_{m_7}C_{m_3}}b\cos(\phi_2)V_1 \\ \phi_2 = \dfrac{1}{R_{m_7}C_{m_3}}V_1 \\ V_1 = \dfrac{R_{m_2}}{R_{m_1}R_{m_3}C_{m_1}}\sin\left(\dfrac{R_{m_5}}{R_{m_4}}\phi_1\right)V \end{cases} \qquad (3)$$

The two memristor exhibits voltage-ampere characteristics and memristor properties.

RESEARCH METHODOLOGY CONT'D

The existing 3D-variable boostable system was modified into a 5D-hyperchaotic system with the introduction of the two memristors in series described in equation (2) and (3) where $W_1(u) = \sin(u)$ and $W_2(w) = \cos(w)$

- The new integer order 5D-memeristive based system is given as:

$$\begin{aligned}
\dot{x} &= \alpha z^2 - 1 \\
\dot{y} &= \alpha z^2 - byz \\
\dot{z} &= x - y^2 + cqy \sin u \cos w \\
\dot{u} &= qy \\
\dot{w} &= qy \sin u
\end{aligned} \qquad (2)$$

where the constant parameters are α = 1.0, β = 6.0, a = 1.0, b = 1.5, c = 1.0 with (1, 1, 0.1, 0.1,0.1) initial conditions Equation (4) is integer system. The dual series memristor in Equation (2) has state variables u and w with sine and cosine trigonometric functions W_1 and W_2 respectively. **The modified 5D system is hyperchaotic.**

RESEARCH METHODOLOGY CONT'D

Synchronization of 5D chaotic memristive systems via active control method

The equations describing the actice control scheme is described below:

$$\begin{cases} \dot{x}_1 = \alpha z_1^2 - 1 \\ \dot{y}_1 = a z_2^2 + b y_1 z_1 \\ \dot{z}_1 = x_1 - y_1^2 + c\beta y_1 \cos w_1 \sin u_1; \\ \dot{u}_1 = \beta y_1 \\ \dot{w}_1 = \beta y_1 \sin u_1 \end{cases}$$

$$\begin{cases} \dot{x}_2 = \alpha z_2^2 - 1 + C_x \\ \dot{y}_2 = a z_2^2 + b y_2 z_2 + C_y \\ \dot{z}_2 = x_2 - y_2^2 + c\beta y_2 \cos w_2 \sin u_2 + C_z; \\ \dot{u}_2 = \beta y_2 + C_u \\ \dot{w}_2 = \beta y_2 \sin u_2 + C_w \end{cases} \qquad (3)$$

- Where $C_{x,}$ $C_{y,}C_{z,}$ $C_{u,}C_{w,}$ are the control functions of the active control scheme used to achieve synchronization between the drive and response.

RESEARCH METHODOLOGY CONT'D

Error Dynamics of the Drive and Response of the System

The error dynamics computed using the drive system, response system and the error system is given by the equation below:

$$\begin{cases} \dot{e}_x = \alpha(e_z^2 + 2e_z z_1) + C_x \\ \dot{e}_y = a(e_z^2 + 2e_z z_1) + b(y_2 z_2 - y_1 z_1) + C_y \\ \dot{e}_z = e_x - (e_y^2 + 2e_y y_1) + (c\alpha y_2 \cos w_2 \sin u_2 - c\beta y_1 \cos w_1 \sin u_1 + C_z) \\ \dot{e}_u = \beta e_y + C_u \\ \dot{e}_w = \beta(y_2 \sin u_2 - y_1 \sin u_2) + C_w \end{cases} \quad (4)$$

RESEARCH METHODOLOGY CONT'D

5D Memristive Based System Electronic Circuit and Arduino link Simulation

The electronic equation is:

$$
\begin{aligned}
\frac{dV_x}{dt} &= \frac{1}{10}\frac{R}{R_1}V_z^2 - 10\frac{R}{R_2} \\
\frac{dV_y}{dt} &= \frac{1}{10}\frac{R}{R_3}V_z^2 + 10\frac{R}{R_4}V_yV_z \\
\frac{dV_z}{dt} &= 10\frac{R}{R_5}V_x - \frac{1}{10}\frac{R}{R_6}V_y^2 + \frac{1}{10}\frac{R}{R_7}V_y\cos(V_w)\sin(V_v) \\
\frac{dV_v}{dt} &= 10\frac{R}{R_8}V_y \\
\frac{dV_w}{dt} &= \frac{1}{10}\frac{R}{R_9}V_z^2V_y\sin(V_v)
\end{aligned}
\tag{5}
$$

where $R = 10k\Omega, R_1 = R_2 = R_3 = R_6 = R_8 = R_9 = 1k\Omega$, $R_4 = 0.66k\Omega$, $R_5 = 100k\Omega$, $R_7 = 16.7k\Omega, V_x, V_y, V_z, V_v$, and V_w, are state variables.

RESEARCH METHODOLOGY CONT'D

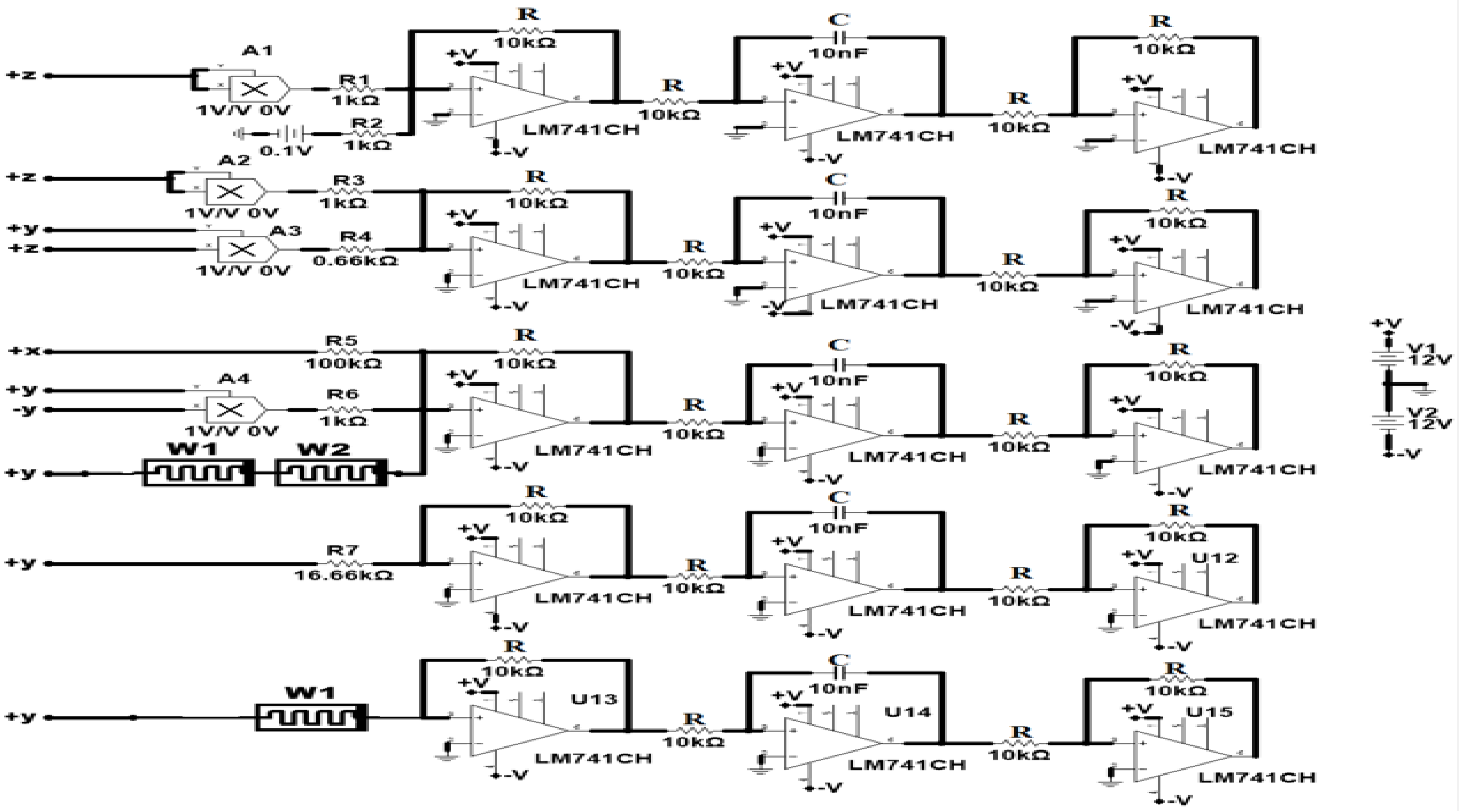


Figure 1: Experimental circuit diagram of the 5D memristive based system

RESEARCH METHODOLOGY CONT'D

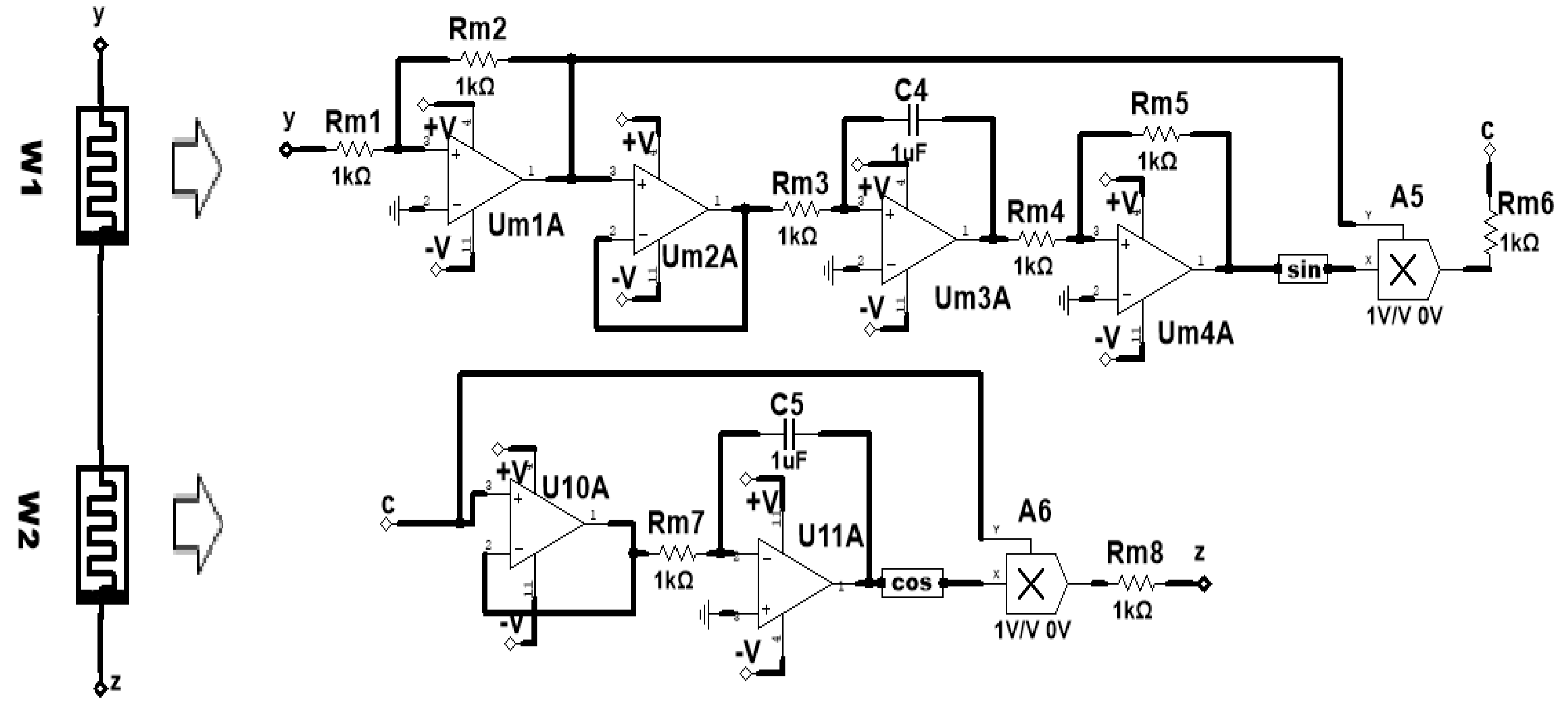


Figure 2: Circuit structure diagram of sine memristor and cosine memristor

RESEARCH METHODOLOGY CONT'D

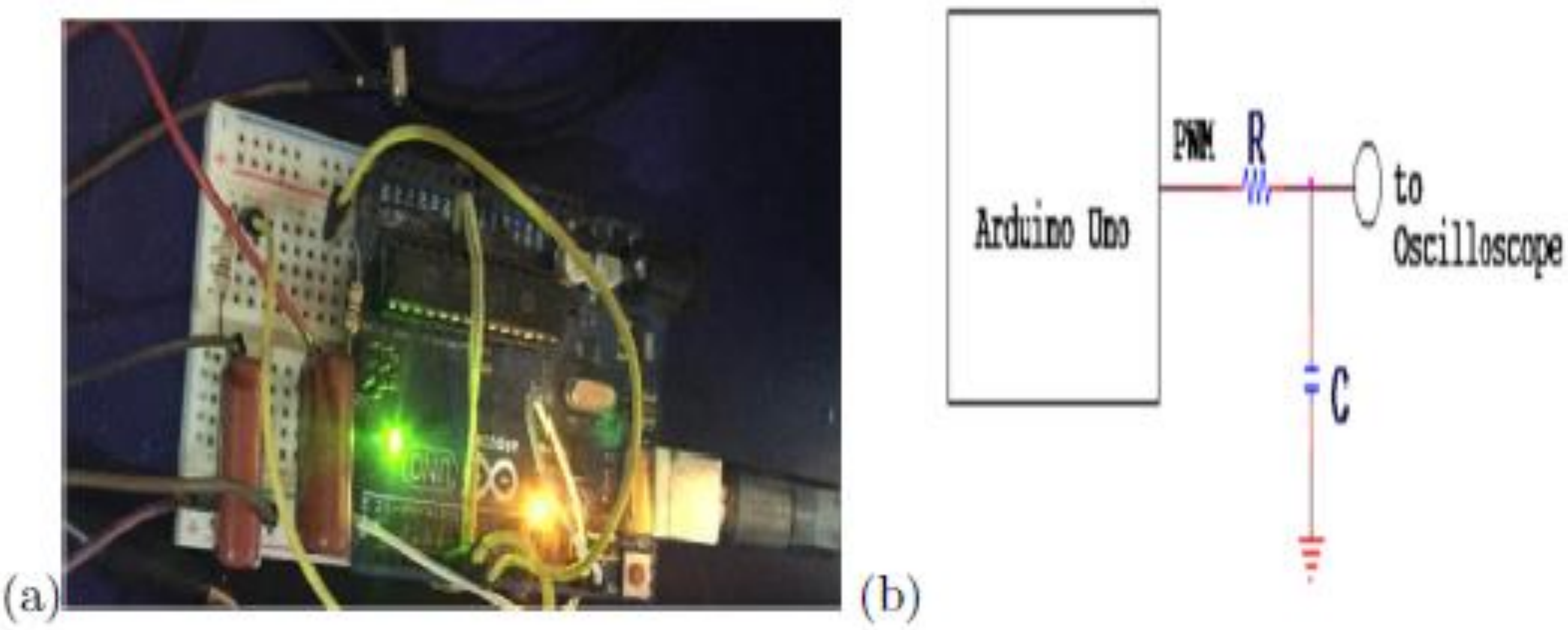


Plate 1: (a) An Arduino set-up (b) Corresponding block diagram

RESULTS AND DISCUSSION

MULTI – EQUILIBRIUM STATES

The following equilibrium points are observed:

i. x = 0, y = 0, z = -1, u = kπ, w(infinitely many due to sin(u) periodicity)
ii. x = 0, y = 0, z = 0, u = kπ, w(infinitely many due to sin(u) periodicity)
iii. x = 0, y = 0, z = 1, u = kπ, w(infinitely many due to sin(u) periodicity)

STABILITY ANALYSIS

- With eigenvalues α equal to 1, the system's equilibrium state is characterized as unstable, indicating **hyperchaotic behavior.**

RESULTS AND DISCUSSION CONT'D

Lyapunov Exponent and Kaplan-Yorke Calculations

The hyperchaotic characteristics of two positive and three negative values are $E_1 = 0.14832, E_2 = 0.00107, E_3 = -0.05147, E_4 = -0 - 02241$ and $E_5 = -1.1343$. Therefore, the Lyapunov dimension is calculated using the Kaplan-Yorke formula given as:

$$D_{KY} = 4 + \frac{1}{|L_5|}\sum_{i=1}^{4} L_i$$

$$= 4 + \frac{0.14832 + 0.00107 - 0.05147 - 0.02241}{|-1.1343|} = 4.0666$$

The positive result corresponds to a positive sum of Lyapunov exponents, indicating chaotic/hyperchaotic behavior.

RESULTS AND DISCUSSION CONT'D

(A) NUMERICAL SIMULATION

- Coexistence of Attractors
- Temporal Variation of the system
- Bifurcation Diagrams and Lyapunov Exponents
- Attractor Phase plots and Time Series
- Results for Environmentally coupled system

RESULTS AND DISCUSSION CONT'D

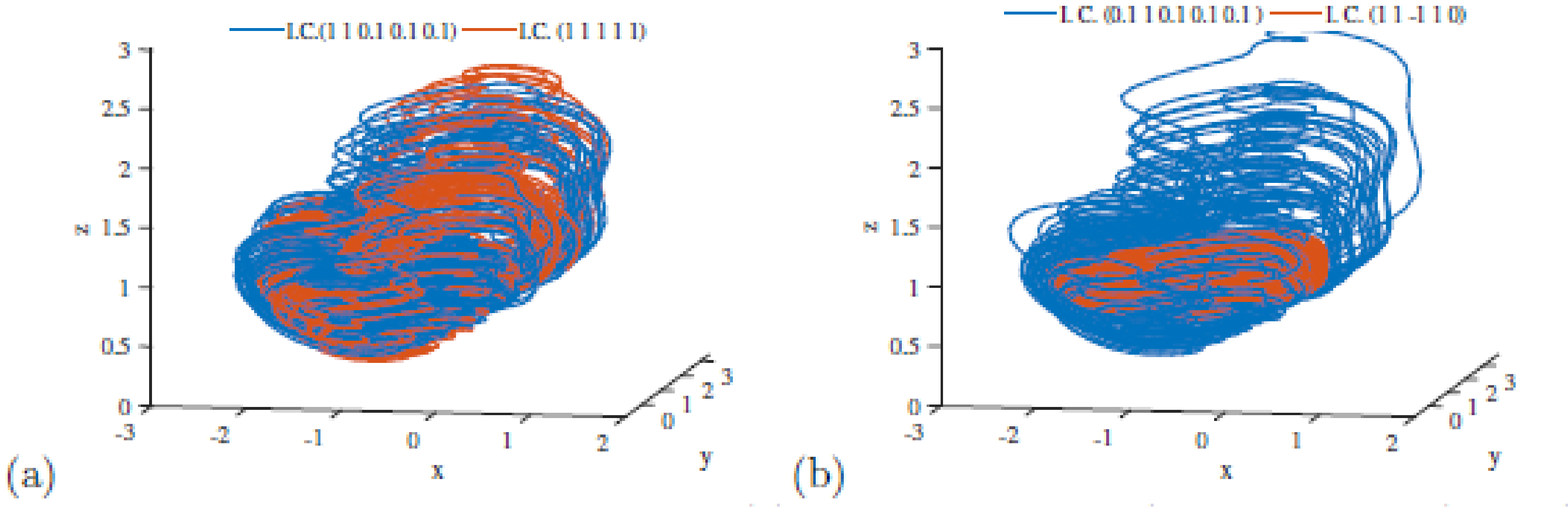


Figure 4 illustrate the coexistence of attractors in the system

Figure 4: Coexistence of Attractors for (a) Initial condition – (0.1 1 0.1 0.1 0.1) and (1 1 1 1 1), (b) (0.1 1 0.1 0.1 0.1) and (1 1 -1 1 0)

RESULTS AND DISCUSSION CONT'D

(a)

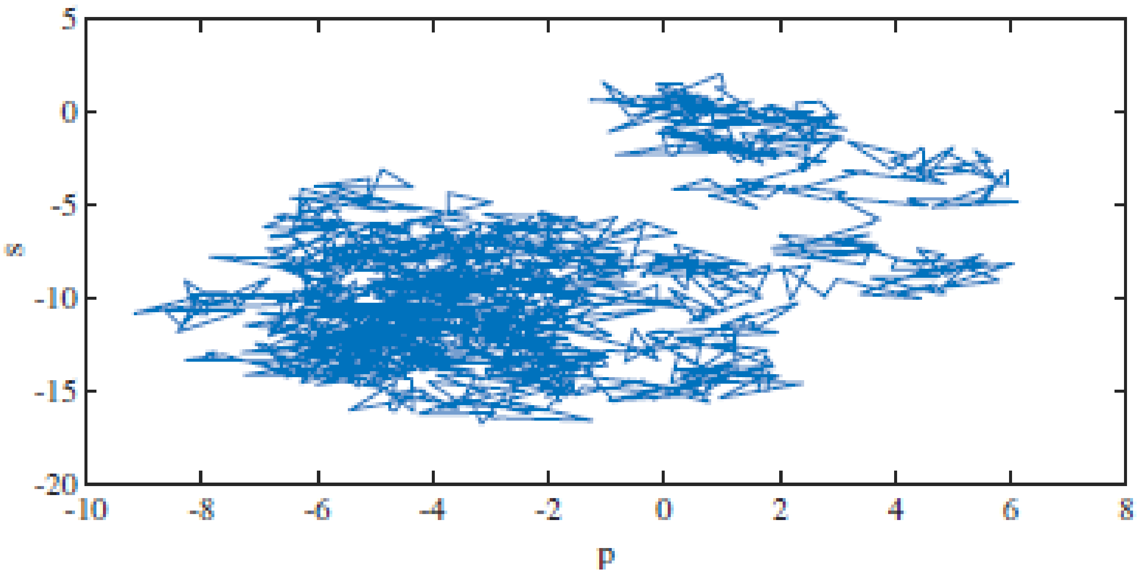


Figure 5 (a) showing the chaoticity behavior

Figure 5 Temporal variation of the (a) s-p plot $a = 1, b = 1.5, c = 1, \beta = 6.$

RESULTS AND DISCUSSION CONT'D

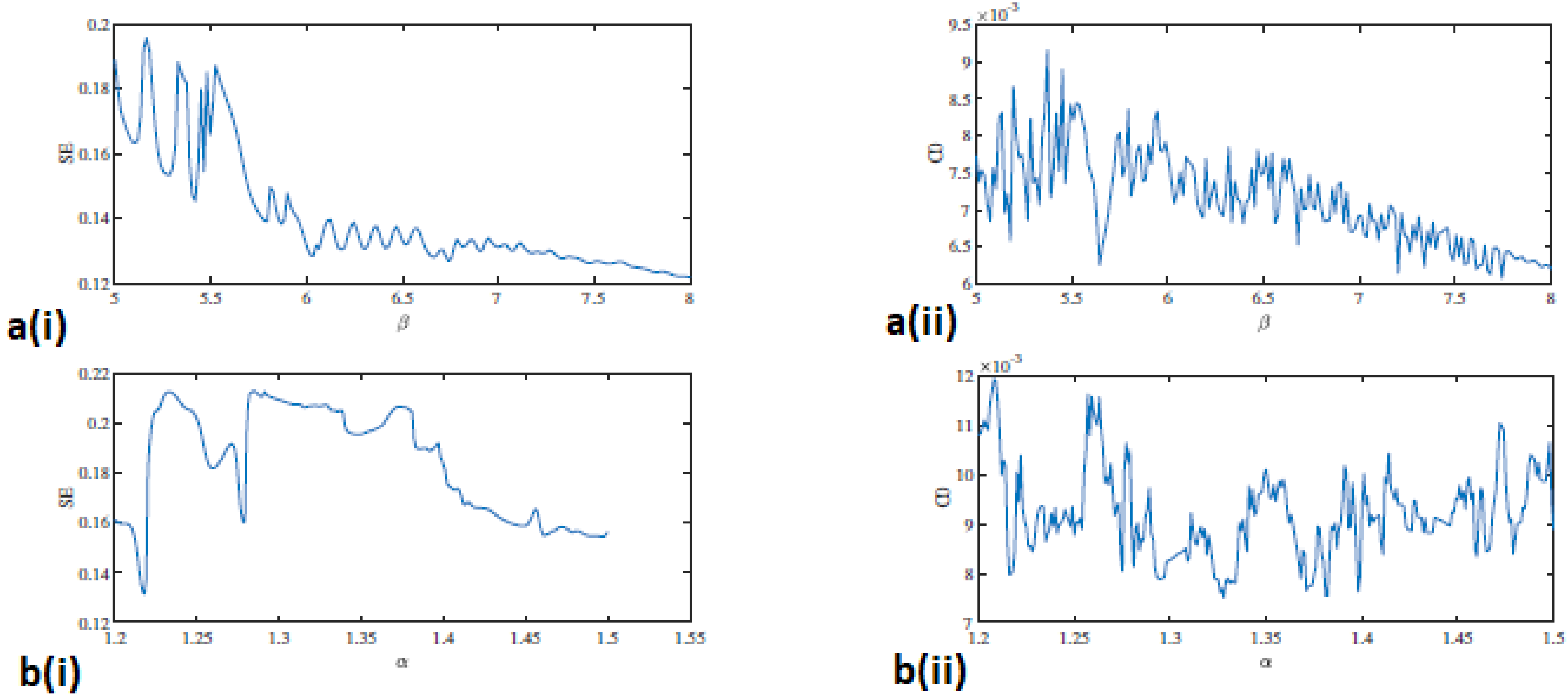


Figure 6: Shannon Entropy (SE) and Complexity (CO) for bifurcation parameters β and α

RESULTS AND DISCUSSION CONT'D

(a)

(b)

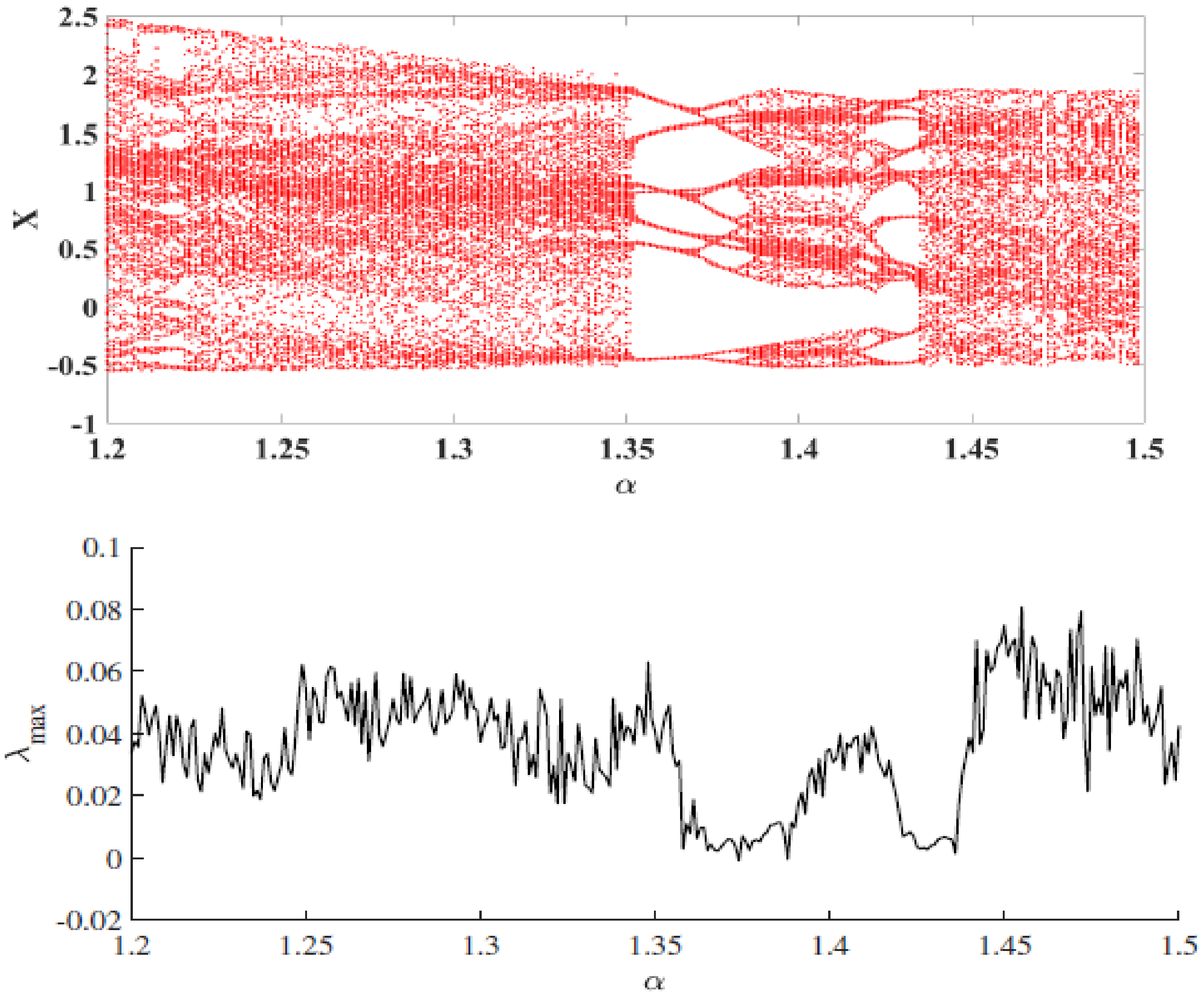


Figure 7: Bifurcation parameters: $a = 1, b = 1.5, c = 1, \beta = 6$
(a) Bifurcation diagram, (b) Corresponding Lyapunov exponent.

In Figure 7 changes to bifurcation parameter α shows chaos between $1.2 \leq \alpha \leq 1.35$ and $1.25 \leq \alpha \leq 1.5$, while reverse periodic cascade occurs between $1.35 \leq \alpha \leq 1.44$

RESULTS AND DISCUSSION CONT'D

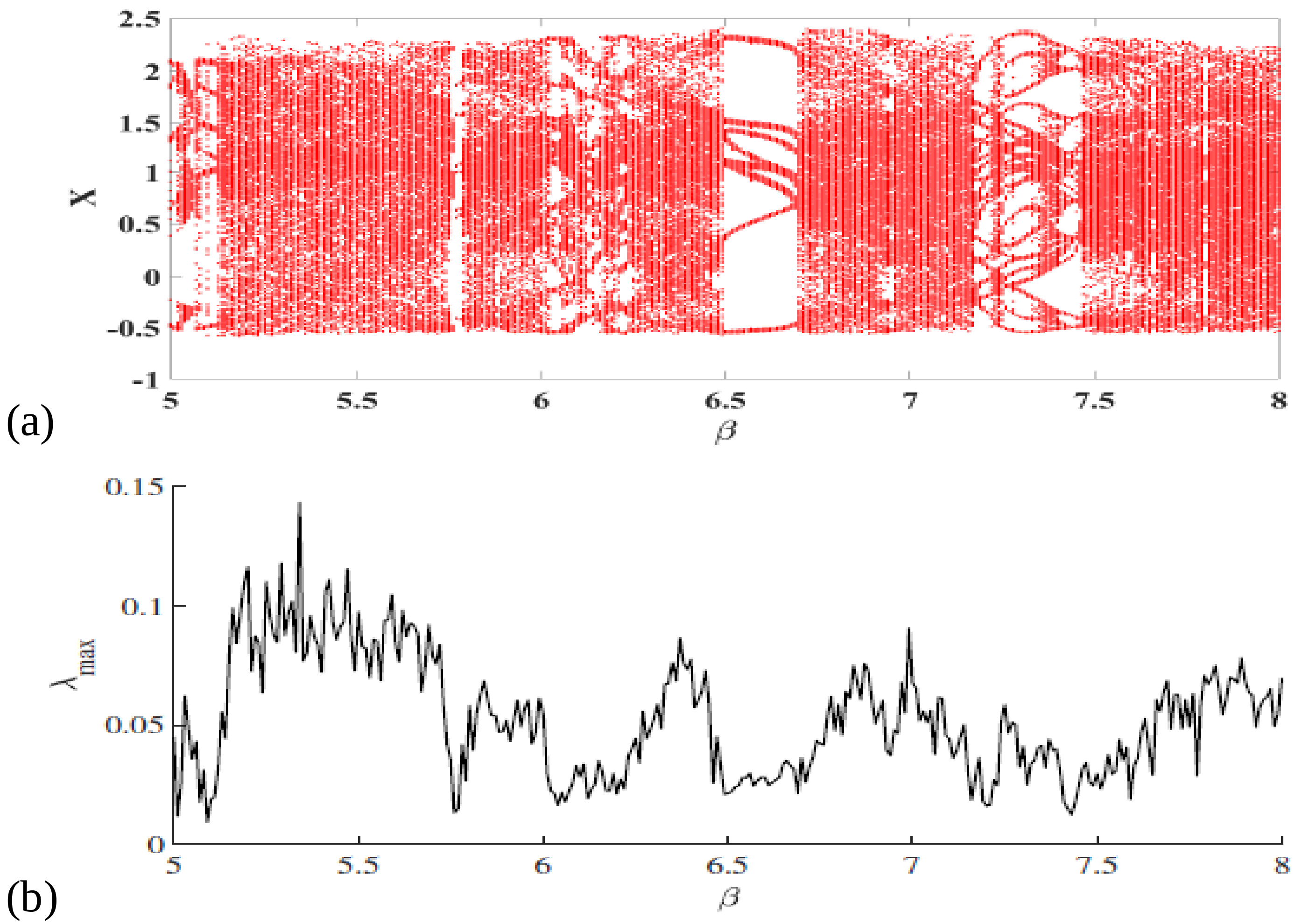
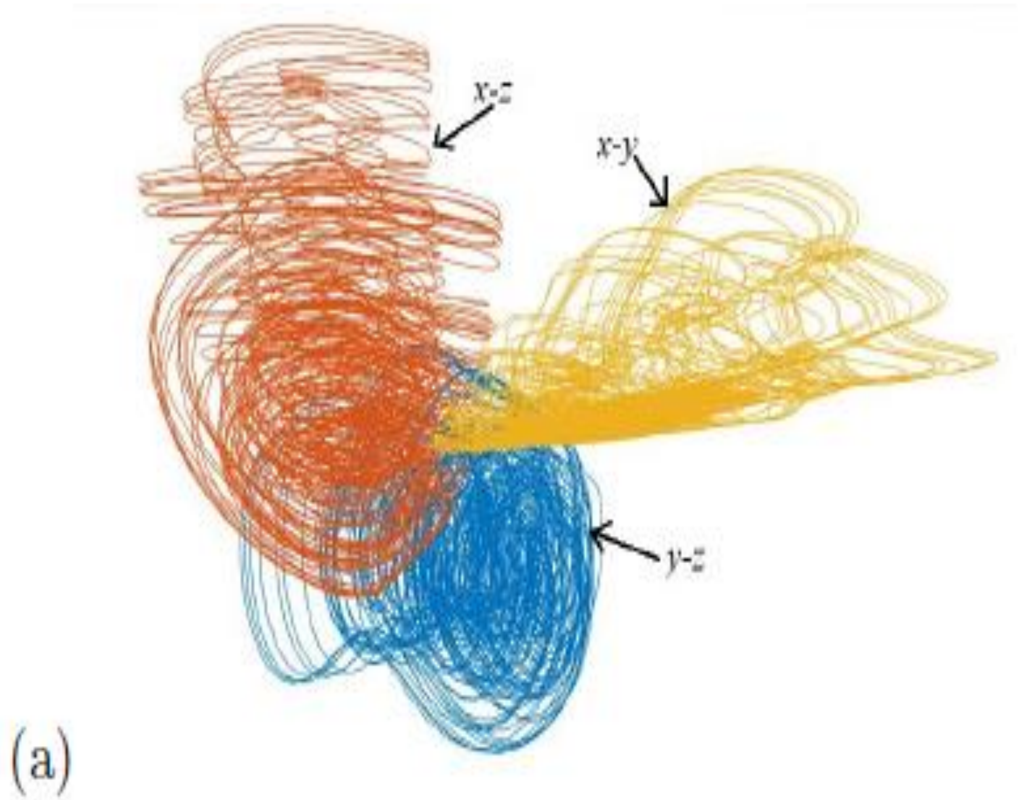


Figure 8: Bifurcation parameters: $a = 1, b = 1.5, c = 1, \alpha = 1.25$ (c) Bifurcation diagram (d) Corresponding Lyapunov exponent.

In Figure 8, the parameter β also reveals a series of behaviors, including chaotic window, Hopf bifurcation, forward periodic cascade, and chaos from selected range $5 \leq \beta \leq 8$.

RESULTS AND DISCUSSION CONT’D



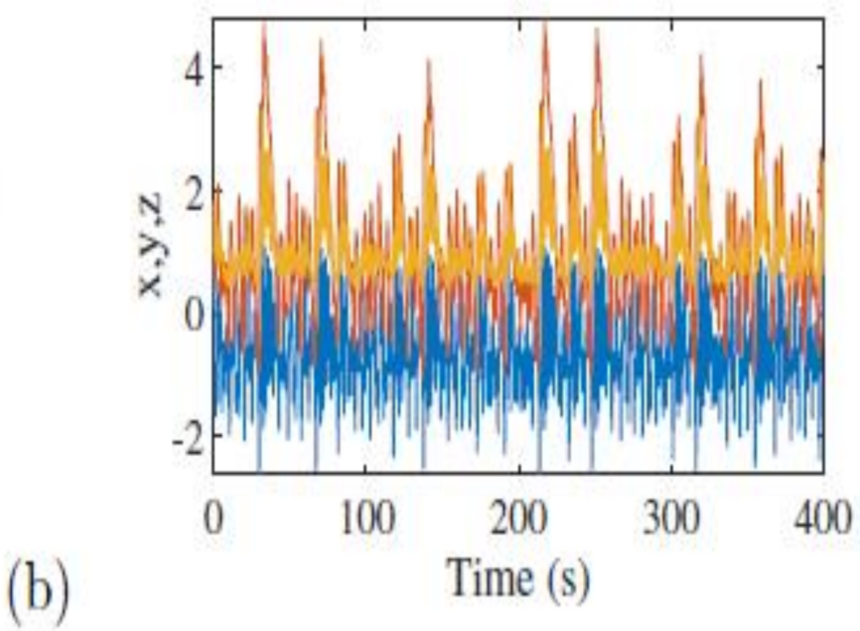


Figure 9: The phase plot for (a) y/x, z/x, and z/y attractors and (b) corresponding time series

RESULTS AND DISCUSSION CONT'D

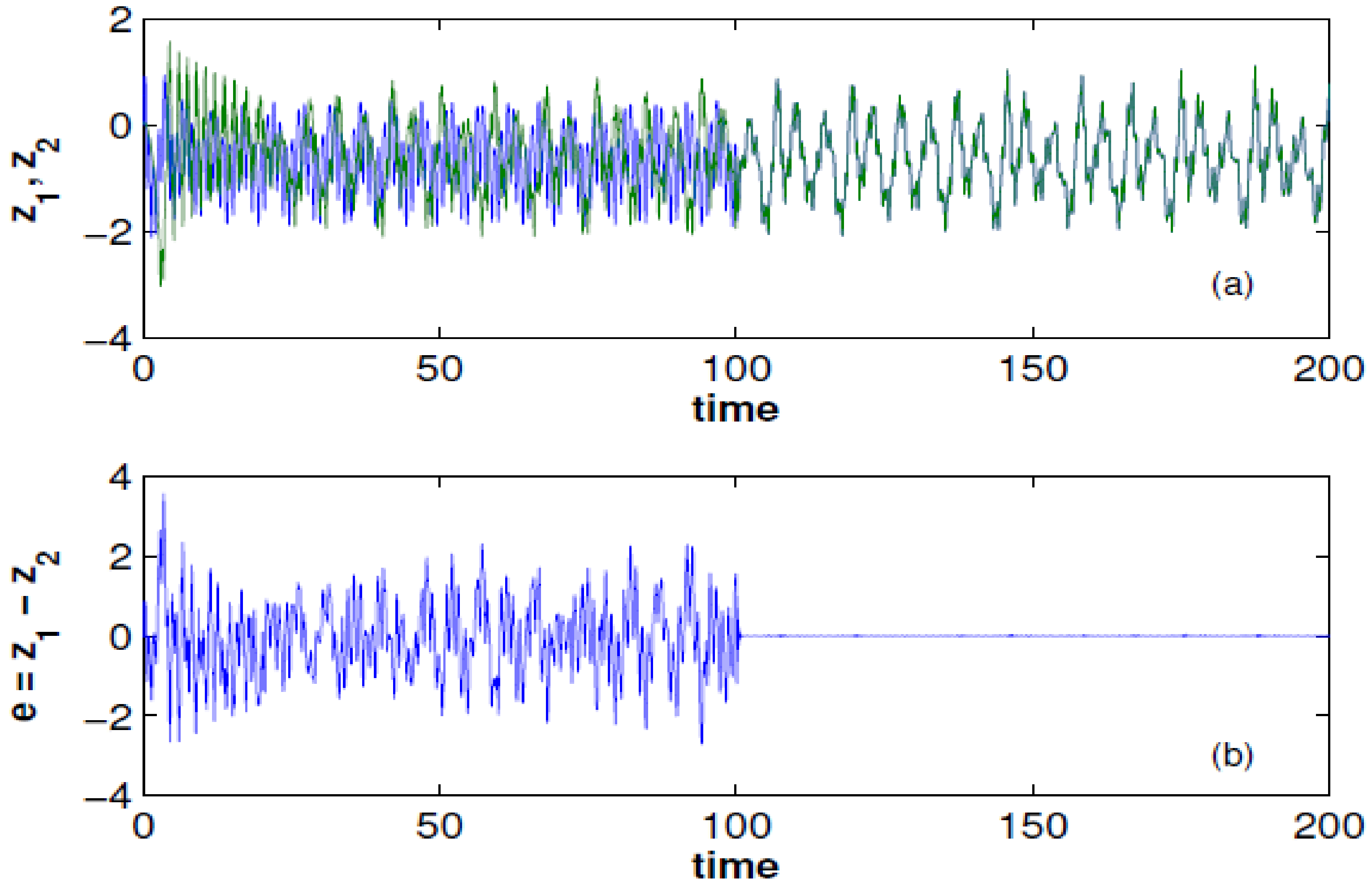


Figure 10: (a) The dynamics of the state variables when the controller was activated at $\boldsymbol{t = 100}$ (b) The dynamics of the error variables when the controller was activated at $\boldsymbol{t = 100}$

Figure 10(a) shows that state variables do not achieve identical dynamics when the controllers are deactivated for $\boldsymbol{0 < t < 100}$ and when the controllers are activated at $t = 80$ the system variables achieve identical dynamics.

Figure 10(b) shows the error dynamics moved chaotically with time when the controllers were deactivated for $\boldsymbol{0 < t < 100}$ while the error system was reduced to zero when the control was applied at $\boldsymbol{t = 100}$.

RESULTS AND DISCUSSION CONT'D

(B) EXPERIMENTAL RESULTS

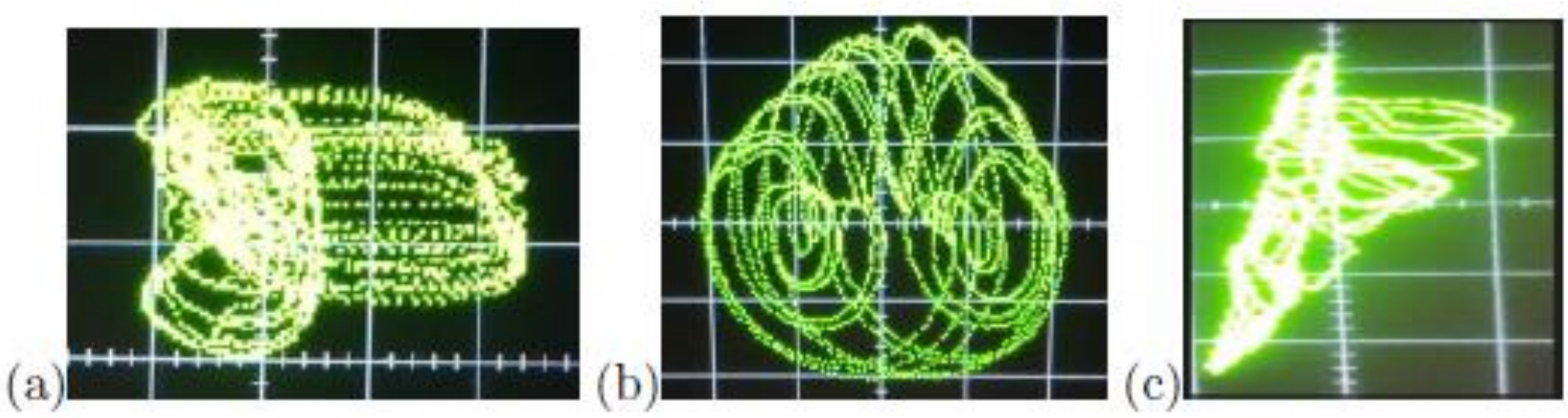


Plate 2: The experimental attractors for (a) $V_x - - V_y, (b)\ V_x - - V_z$, **and** (c) $V_y - - V_z$

RESULTS AND DISCUSSION CONT'D

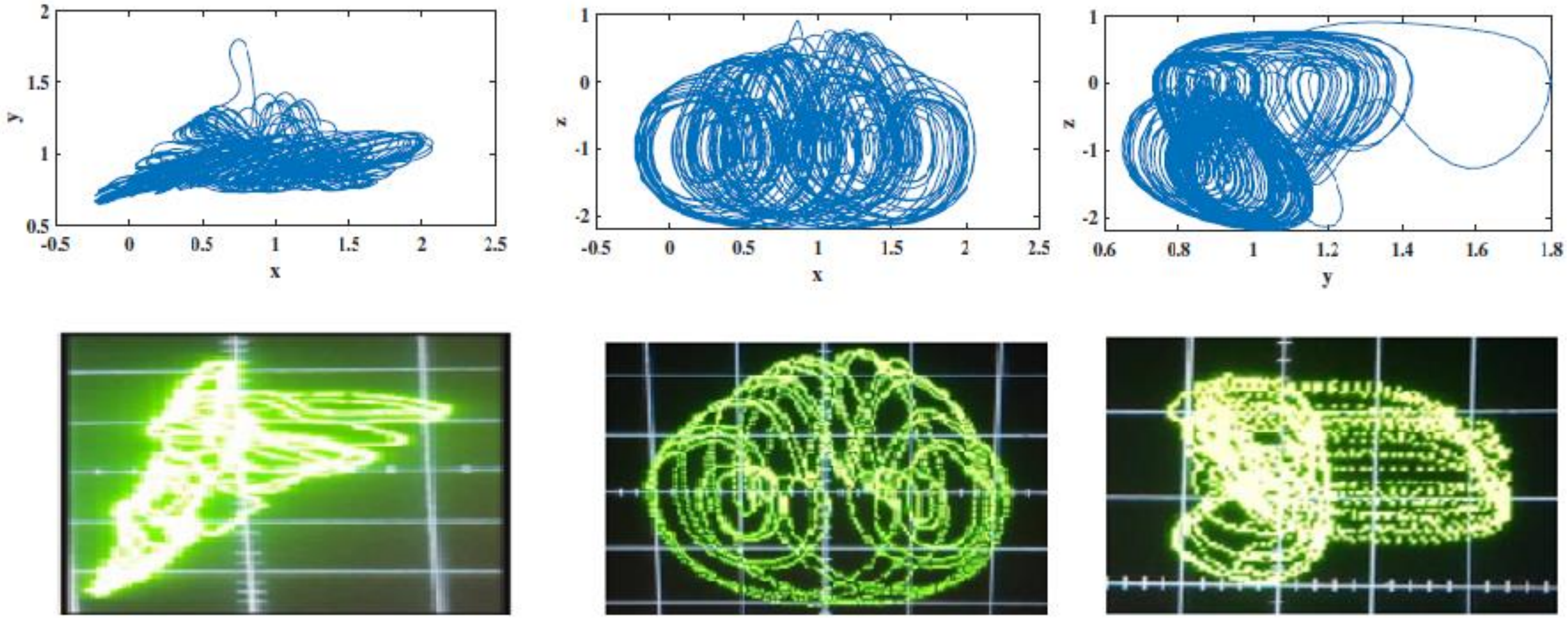


Figure 11: Numerical versus experimental results

RESULTS AND DISCUSSION CONT’D

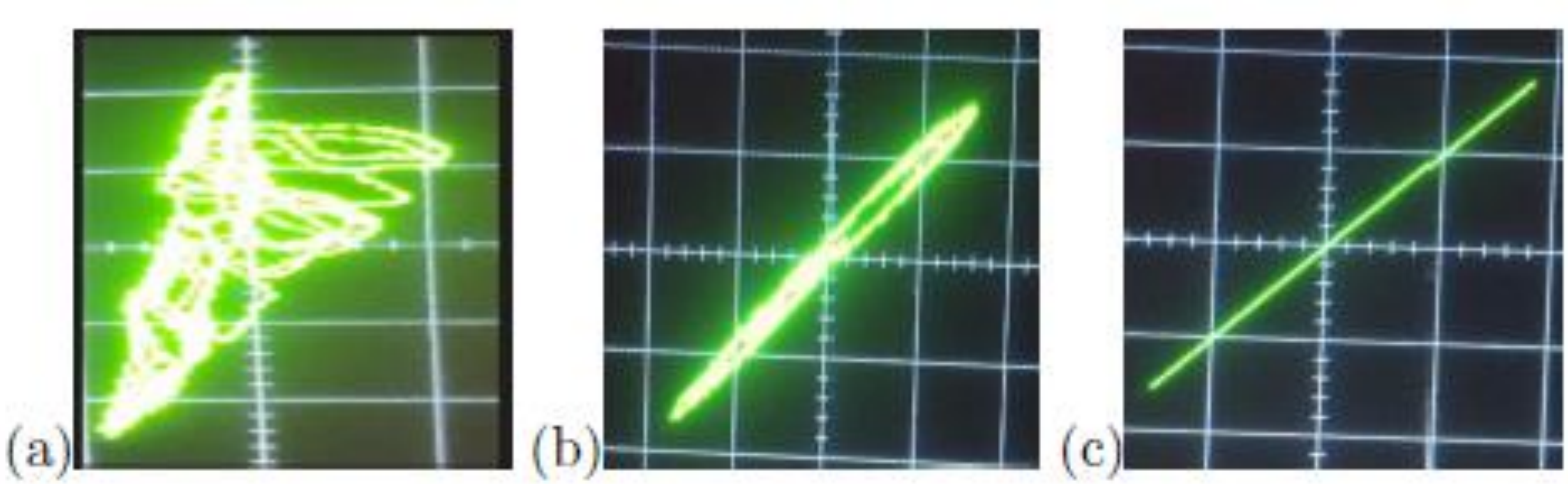


Plate 3: The results show (a) without coupling, (b) incomplete synchronization, and (c) complete synchronization when varying the coupling strength

CONTRIBUTIONS TO KNOWLEDGE

This study has been able to:

- establish a new multiequilibrium variable boostable hyperchaotic system by introducing a double-based memristors.
- achieve global synchronization using active control scheme numerically and experimentally.
- Paved the way for applications across various fields by showcasing the adaptability and performance enhancements of MSVB systems.

CONCLUSION

- **Analyzed 5D variable-boostable system:** Investigated hyperchaotic behaviors transitioning from 3D chaotic to 5D hyperchaotic flows, with detailed stability and behavior analyses.

- **Introduced Active control scheme on the MSVB system:** Presented adaptability and performance enhancements through numerical simulations and practical Arduino-based implementations.

- **Combined theoretical analyses with real-world demonstrations:** Bridged theoretical insights with practical applications, offering valuable understanding of 5D system dynamics.

RECOMMENDATION

- Dynamical coupling scheme can further be applied to study ecological system.

- Further application of the proposed system to secure-communication, image encryption e.t.c.

- Collaborate with industry partners or research institution for integration into relevant technologies.

THANK
YOU
FOR LISTENING