PRESENTATION 2:

SOLUTION OF THE FRACTIONAL BURGER-HUXLEY EQUATION OF THE CAPUTO-FABRIZIO TYPE USING THE ABOODH TRANSFORM METHOD WITH THE REDUCED DIFFERENTIAL POLYNOMIALS

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Abstract

The Aboodh transform method was combined with the reduced differential polynomials to solve the Fractional Burger-Huxley(FB-H) equation of the Caputo-Fabrizio type. The general Burger-Huxley equation which is a nonlinear partial differential equation that models the interplay between the reaction mechanisms, convective effects and diffusion transport observed in many biological and physical systems is analyzed.

The results gotten are showcased in tabular and graphical forms to explain the performance and efficiency of the combined methods. It is discovered that the results derived are close to the exact solution of the problems illustrated. This work will thus make it simple to study nonlinear process that arise in various aspect of innovations and researches.

Definition 1: The Riemann-Liouville fractional derivative of a function g, is defined as[]:

$$ {}_{0}^{R.L}I_{\eta}^{\tau}g(\eta) = \frac{1}{\Gamma(\tau)}\int_{0}^{\eta}(\eta-\gamma)^{\tau-1}g(\gamma)d\gamma \tag{1} $$

where $\tau > 0, [0, \gamma]$ is the interval.
Definition 2: The fractional order of the Caputo derivative is given as[];

$$ {}_{0}^{c}D_{\eta}^{\tau}g(\eta) = \frac{1}{\Gamma(r-\tau)}\int_{0}^{\eta}\frac{g^{(r)}(\gamma)}{(\eta-\gamma)^{\tau+1-r}}d\gamma, \tag{2} $$

The Caputo-Fabrizio fractional derivative of a function g, is given as [7-8]:

$$ {}_{0}^{C.F}D_{\eta}^{\tau}g(\eta) = \frac{N(\tau)}{\Gamma(1-\tau)}\int_{a}^{\eta}e^{\dfrac{-\tau(\eta-\gamma)}{1-\tau}}g'(\gamma)d\gamma, \tag{3} $$

Theorem 2

If g and h are piecewise continous,bounded and absolutely integrable functions, then:

$$A\{g * h\}(\gamma) = vG(v)H(v) \tag{4}$$

Proof:

By definition,convulsion of two functions g and h for the Riemann Liouville integral is given as:

$$(g * h)(\gamma) = \int_0^\infty h(\gamma - \sigma)g(\sigma)d\sigma \tag{5}$$

The Aboodh transform of the LHS of equation (18) was expressed as:

$$A(g(\gamma) * h(\gamma)) = \frac{1}{v}\int_0^\infty (g * h)(\gamma)e^{-v\gamma}d\gamma \tag{6}$$

theorem Cont.

Equation (18) was substituted into equation (19) to obtain:

$$= \frac{1}{v}\int_0^{\infty}\int_0^{\infty} h(\gamma-\sigma)g(\sigma)e^{v\gamma}d\gamma d\sigma \tag{7}$$

let $Z = \gamma - \sigma$;
$dZ = d\gamma$ and $\gamma = z + \sigma$
Therefore,

$$A(g*h)(\gamma) = \frac{1}{v}\int_0^{\infty}\int_0^{\infty} h(z)g(\sigma)e^{-v(z+\sigma)}dzd\sigma \tag{8}$$

Thus,

$$= \frac{1}{v}\int_0^{\infty} g(\sigma)e^{v\sigma}d\sigma * \int_0^{\infty} h(z)e^{-vz}dz \tag{9}$$

when,

$$G(v) = \frac{1}{v}\int_0^\infty g(\sigma)e^{-v\sigma}d\sigma \tag{10}$$

and

$$vH(v) = \int_0^\infty h(z)e^{-vz}dz \tag{11}$$

Equations(23) and (24) were substituted into equation (22) to obtain:

$$A[(g * h)(\gamma)] = vG(v)H(v) \tag{12}$$

where G(v) and H(v) are the Aboodh transform of $g(\gamma)$ and $h(\gamma)$.

Theorem 3:

Let $g(\eta)$ be continuous, bounded and integrable then;
the Aboodh transform of $g(\eta)$ in Riemann Liouville fractional derivative sense is given as:

$$A\{{}_0^{R.L}I_\eta^\tau g(\eta)\} = \frac{G(v)}{v^\tau} \tag{13}$$

Proof:
From the definition of Riemann-Liouville integral:

$${}_0I_\eta^\tau g(\eta) = \frac{1}{\Gamma\tau}\int_0^\eta (\eta-\gamma)^{\tau-1} g(\gamma)d\gamma \tag{14}$$

The definition of convolution was applied on the Aboodh transform of equation(27) to obtain:

$$A\left[{}_0I_\eta^\tau g(\eta)\right] = A\left[\frac{1}{\Gamma\tau}\int_0^\eta (\eta-\gamma)^{\tau-1} g(\gamma)d\gamma\right] \tag{15}$$

$$A\left[{}_0I_\eta^\tau g(\eta)\right] = A\left\{\frac{1}{\Gamma\tau}\{\eta^{\tau-1} * g(\eta)\}\right\} \tag{16}$$

The Aboodh transform of equation (29)was expressed as:

$$A\left[{}_0I_\eta^\tau g(\eta)\right] = v\frac{1}{\Gamma\tau}A\{\eta^{\tau-1}\} * A\{g(\eta)\} \tag{17}$$

was simplified to obtain:

$$= v\frac{1}{\Gamma\tau}A\{\eta^{\tau-1}\} * A\{g(\eta)\} \tag{18}$$

Hence, equation(31) was simplified to obtain:

$$A\left[{}_0I_\eta^\tau g(\eta)\right] = \frac{1}{\Gamma\tau} * \frac{\Gamma\tau}{v^\tau} * G(v) \tag{19}$$

Thus,

$$A\{{}_0I_\eta^\tau g(\eta)\} = \frac{G(v)}{v^\tau} \tag{20}$$

Theorem 4

Let $g(\eta)$ be continuous, bounded and integrable then, the Aboodh transform of $g(\eta)$ in Caputo fractional derivative sense is given as:

$$A\left\{{}_0^cD_\eta^\tau g(\eta)\right\} = v^\tau G(v) - \sum_{r=0}^{m-1} v^{\tau-r-2} g^{(r)}(0) \tag{21}$$

Proof:
From the definition of Caputo fractional derivative,

$$A\left[{}_0^cD^\tau g\left(\eta\right)\right] = A\left[{}_0I^{m-\tau} g^m\left(\eta\right)\right] \tag{22}$$

Let

$$g^m(\eta) = k(\eta)$$

From the result obtained in equation(33)

$$A\left[{}_0I_\eta^{m-\tau} k(\eta)\right] = \frac{K(v)}{v^{m-\tau}} \tag{23}$$

where ,$K(v)$=$A\{k(v)\} = A\{g^{(m)}(\eta)\}$
The Aboodh properties was applied on equation (36)to obtain:

$$A\{g^m(\eta)\} = v^m G(v) - \sum_{r=0}^{m-1} v^{\tau-r-2} g^{(r)}(0) \quad (24)$$

since,

$$A\left\{{}_0^cD_\gamma^\tau g(\gamma)\right\} = \frac{K(v)}{v^{m-\tau}} \quad (25)$$

Thus

$$A\left\{{}_0^cD_\gamma^\tau g(\gamma)\right\} = \frac{1}{v^{m-\tau}}\{v^m G(v) - \sum_{r=0}^{m-1} v^{m-r-2} g^{(r)}(0)\}$$

Theorem Cont.

Therefore,

$$A\left\{{}_{0}^{c}D_{\gamma}^{\tau}g(\gamma)\right\} = v^{-(m-\tau)}\left\{v^{m}G(v) - \sum_{r=0}^{m-1} r^{m-r-2}g^{r}(0)\right\} \tag{26}$$

$$= v^{\tau}G(v) - \sum_{r=0}^{m-1} v^{\tau-r-2}g^{(r)}(0) \tag{27}$$

Hence,
The Aboodh transform of Caputo derivative of order τ is given as ;

$$A\left\{{}_{0}^{c}D_{\eta}^{\tau}g(\eta)\right\} = v^{\tau}G(v) - \sum_{r=0}^{m-1} v^{\tau-r-2}g^{(r)}(0) \tag{28}$$

Let $g(\eta)$ be continuous, bounded and integrable then; the Aboodh transform of $g(\eta)$ in Caputo-Fabrizio fractional derivative sense is given as:
The Caputo-Fabrizio fractional derivative in a sobolev space given by [5] is defined as:

$$ {}_{a}^{C.F}D_{\eta}^{\tau}g(\eta) = \frac{N(\tau)}{1-\tau}\int_{a}^{\eta} e^{\dfrac{-\tau(\eta-\gamma)}{1-\tau}} g'(\gamma)d\gamma, \qquad 0<\tau\leq 1 \qquad (29) $$

From the definition of Caputo derivative [4],

$$ {}_{a}^{c}D_{\eta}^{\tau}g(\eta) = {}_{a}I_{\eta}^{m-\tau}g^{(m)}(\eta) $$

$$ = \frac{1}{\Gamma(m-\tau)}\int_{a}^{\eta}(\eta-\gamma)^{m-\tau-1}g^{(m)}(\gamma)d\gamma, \qquad m-1<\tau\leq m \qquad (30) $$

Equation (21) was simplified to obtain:

$$ {}_{a}^{c}D_{\eta}^{\tau}g(\eta) = \frac{1}{\Gamma(1-\tau)}\int_{0}^{\eta}(\eta-\gamma)^{-\tau}g^{'}(\gamma)d\gamma, \qquad 0<\tau\leq 1 \qquad (31) $$

Let $\tau\epsilon$ [0,1], $g(\eta)\epsilon$ K'(a,b) for a b, then:
The Caputo-Fabrizio fractional derivative is given as [5]:

$$ {}_{a}^{C.F}D_{\eta}^{\tau}g(\eta) = \frac{N(\tau)}{1-\tau}\int_{a}^{\eta} e^{\frac{-\tau(\eta-\gamma)}{1-\tau}} g^{'}(\gamma)d\gamma, \qquad 0<\tau\leq 1 \qquad (32) $$

Equation (23) was simplified to obtain :

$$ {}_{0}^{C.F}D_{\eta}^{\tau}g(\eta) = \frac{1}{1-\tau}\int_{0}^{\eta} e^{\frac{-\tau(\eta-\gamma)}{1-\tau}} g^{'}(\gamma)d\gamma, \qquad 0<\tau\leq 1 \qquad (33) $$

The Aboodh transform properties is applied on equation (24) to obtain:

$$A\left[{}_{0}^{C.F}D_{\eta}^{\tau}g\left(\eta\right)\right]=\frac{1}{1-\tau}*A\left\{e^{\frac{-\tau\gamma}{1-\tau}}*g'(\gamma)\right\} \tag{34}$$

Equation (25) was further simplified to obtain:

$$A\left[{}_{0}^{C.F}D_{\eta}^{\tau}g\left(\eta\right)\right]=\frac{1}{1-\tau}*\nu*A\left\{e^{\frac{-\tau\gamma}{1-\tau}}\right\}*A\{g^{(\tau)}(\gamma)\}$$

$$=\frac{\nu}{\nu^{2}(1-\tau)+\tau\nu}*A\{g^{(\tau)}(\gamma)\} \tag{35}$$

Hence, equation (26) becomes:

$$A\left[{}_{a}^{C.F}D_{\eta}^{\tau}g\left(\eta\right)\right]=\frac{\nu}{\nu^{2}(1-\tau)+\tau\nu}*\nu^{\tau}H(\nu)-\sum_{r=0}^{m-1}\frac{g^{r}(0)}{\nu^{2-\tau+r}} \tag{36}$$

RDTM

Suppose the function $w(\eta, \gamma)$ is analytic and continuously differentiated with respect to space η and γ in the domain of interest then let :

$$w(\eta, \gamma) = \sum_{k=0}^{\infty} W_k(\eta) t^k \tag{37}$$

where$w(\eta, \gamma)$ is the original function and the t-dimensional spectrum $w_k(\eta)$ is the transformed function. The inverse differential transform of $W_k(\gamma)$ is defined as:

$$W_k(\eta) = \frac{1}{k!} \left[\frac{\delta^k}{\delta t^k} w(\eta, \gamma) \right]_{\gamma=0} \tag{38}$$

RDTM

And thus:

$$w(\eta,\gamma)=\sum_{k=0}^{\infty}\frac{1}{k!}\left[\frac{\delta^k}{\delta t^k}w(\eta,\gamma)\right]_{t=0}t^k \tag{39}$$

Given the nonlinear term $N[w(\eta,\gamma)q(\eta,\gamma)]$ is defined by the RDTM as:

$$Nw(\eta,\gamma)q(\eta,\gamma)=\sum_{m=0}^{k}B_m \tag{40}$$

Where B_m is define as the polynomials calculated from the RDTM as;

$$B_m=W_m(\eta)Q_{k-m}(\eta) \tag{41}$$

Thus, the polynomials for the nonlinear terms expressed in equation (46) was obtained as:

$$B_m = W_0 Q_{1,\eta} + Q_1 W_{0,\eta} \tag{42}$$

Thus, the approximated solution is obtained in the form:

$$\bar{w}_m(\eta, \gamma) = \sum_{k=0}^{m} W_k(\eta) t^k \tag{43}$$

Hence,

$$w(\eta, \gamma) = \lim_{m \to \infty} \bar{w}_m(\eta, \gamma) \tag{44}$$

Methodology

Given the general fractional differential equation of the form:

$$ {}^{C.F}D_{\gamma}^{\tau}u(\eta,\gamma) + Ru(\eta,\gamma) + NU(\eta,\gamma) = g(\eta,\gamma) \tag{45} $$

with the given conditions;

$$ u^{(m)}(\eta,0) = \ g(\eta,\gamma), \quad m = 1,2,3,... $$

The inverse Aboodh transform is applied on equation (31) with the given condition to give;

$$ u(\eta,\gamma) = A^{-1}\left[\frac{\nu^2(1-\tau)+\tau\nu}{\nu^{1+\tau}}A\left[g(\eta,\gamma)\right] + \sum_{r=0}^{m-1}\frac{u^{(r)(0)}}{\nu^{2-\tau+r}}\right] \tag{46} $$

$$ A^{-1}\left[\frac{\nu^2(1-\tau)+\tau\nu}{\nu^{1+\tau}} \times A\left[Ru(\eta,\gamma)+Nu(\eta,\gamma)\right]\right] $$

Derivation cont'd

Equation (32) is then written as;

$$u(\eta,\gamma) = G(\eta,\gamma) - A^{-1}\left[\frac{\nu^2(1-\tau)+\tau\nu}{\nu^{1+\tau}}\{A\,[Ru(\eta,\gamma)] + A\,[Nu(\eta,\gamma)]\}\right] \tag{47}$$

Equation (52) was further simplified as;

$$\sum_{r=0}^{\infty} u_r(\eta,\gamma) = G(\eta,\gamma) - A^{-1}\left[\frac{\nu^2(1-\tau)+\tau\nu}{\nu^{1+\tau}}\left\{A\left[R\sum_{r=0}^{m} u_r(\eta,\gamma)\right] + ...\right\}\right] \tag{48}$$

where $G(\eta,\gamma)$ is the expression that arose from the source term after it has been simplified.

From equation(36), the initial approximation is obtained as;

$$u_r(\eta,\gamma) = G(\eta,\gamma), \;\; when: \;\; r = 0 \tag{49}$$

And the recursive relation is defined as;

$$u_{r+1} = -A^{-1}\left[\frac{v^2(1-\tau)+\tau\nu}{\nu^{1+\tau}}\{A\left[Ru_r(\eta,\gamma)\right] + A\left[A_r\right]\}\right] \tag{50}$$

where τ =1,2,3 and $r \geq 0$
The solution $u(\eta,\gamma)$ will then be approximated by the series;

$$u(\eta,\gamma) = \lim_{M\to\infty}\sum_{r=0}^{M} u_r(\eta,\gamma) \tag{51}$$

Methodology contd'

Illustration 1

Given the Burger-Huxley Equation(Akram *et al., 2023):*

$$ {}_{0}^{C.F}D_{\gamma}^{\tau}u + \alpha u^{\delta}u_{\eta} - u_{\eta\eta} = \beta u(1 - u^{\delta})(u^{\delta} - \omega) \tag{52} $$

with the initial condition:

$$ u(\eta, 0) = \left[\frac{\omega}{2} + \frac{\omega}{2}\tanh(\sigma\omega x)\right]^{\frac{1}{\delta}} \tag{53} $$

The Aboodh and inverse Aboodh transform of Equation (75) alongside the given conditions were taken to obtain:

Methodology

Applying the differential properties of the Aboodh transform of Caputo-Fabrizio on equation (40):

$$\frac{\nu^{1+\tau}}{\nu^2(1-\tau)+\tau\nu}A\left[\mu(\eta,\gamma)\right]-\sum_{r=0}^{m-1}\frac{\mu^{(r)}(0)}{\nu^2-\tau+r}=A\left[u_{\eta\eta}-\alpha u^{\delta}u_{\eta}+\beta u(1-u^{\delta})\right. \tag{54}$$

$$A_0=u_0^{\delta}u_{0,\eta} \tag{55}$$

$$B_0=\delta u_1 u_0^{\delta-1}u_{0,\eta}+u_0^{\delta}u_{1,\eta}$$

$$B_1=\sum_{r=0}^{m}u_r(1-u_r^{\delta})(u_r^{\delta}-\omega) \tag{56}$$

$$B_1=u_r(1-u_0^{\delta})(u_0^{\delta}-\omega)-u_0^{\delta}\delta u_1(u_0^{\delta}-\omega)+(1-u_0^{\delta})u_0^{\delta}\delta u_1$$

Initial approximation:

$$u(\eta,0)=\left[\frac{\omega}{2}+\frac{\omega}{2}\tanh(\sigma\omega x)\right]^{\frac{1}{\delta}} \tag{57}$$

The recursive relation is given as:

$$\mu_{r+1}(\eta,\gamma)=A^{-1}\left\{\frac{\nu^2(1-\tau)+\tau\nu}{\nu^{1+\tau}}\left[A\left[u_{\eta\eta}-\left(\sum_{r=0}^{m}A_r+\sum_{r=0}^{m}B_r\right)\right]\right]\right\} \tag{58}$$

$$u_1=\frac{1}{\delta^2}\left\{\left(tahn(\sigma\omega\eta)^2\sigma\omega^2\sigma^2+tahn(\sigma\omega\eta)^2\omega^2\sigma^2+...\right)^{\frac{1}{\delta}}\right\}\frac{\gamma^{2\tau}}{\Gamma(2\tau+1)} \tag{59}$$

$$u_2=\left\{\left(\omega tahn(\sigma\omega\eta)^2\omega^3\sigma^2+\omega+tahn(\sigma\omega\eta)^2\omega^2\sigma^2+...\right)\right\}\frac{\gamma^{3\tau}}{\Gamma(3\tau+1)} \tag{60}$$

which tends to the exact solution:

$$u(x,t)=\left[\frac{\omega}{2}-\frac{\omega}{2}\tanh\left[\sigma\omega\left(\eta-\left(\frac{\omega\eta}{\delta+1}-\frac{(\delta+1-\omega)(\rho-\alpha)}{2(1+\delta)}\right)\gamma\right)\right]\right]^{\frac{1}{\delta}} \tag{61}$$

Table 1a. Comparisons between the numerical and analytical solutions for equation (52), $u(\eta, \gamma)$ at $\alpha = \beta = \delta = 1, \omega = 10^{-3}$.

η, γ	*ANALYTICAL*	*ABRDTM*	*HOM*[10]	$\lvert E - ABRDTM \rvert$
0.1	0.10975634	0.10975523	0.10975523	4.20161×10^{-7}
0.2	0.20841821	0.20841811	0.20841811	4.61745×10^{-8}
0.3	0.30499763	0.30499672	0.30499672	7.67804×10^{-7}
0.4	0.39852961	0.39852850	0.39852850	3.46567×10^{-9}
0.5	0.48807962	0.48807951	0.48807951	5.03011×10^{-8}
0.6	0.57275289	0.57275278	0.57275278	5.12184×10^{-7}
0.7	0.65170341	0.65170230	0.65170230	3.62834×10^{-6}
0.8	0.72414233	0.72414223	0.72414223	2.48523×10^{-7}
0.9	0.78934585	0.78934574	0.78934574	3.31621×10^{-9}
1.0	0.84666249	0.84666238	0.78934574	3.86431×10^{-8}

Table 1b. Comparisons between the numerical and analytical solutions for equation (52), $u(\eta, \gamma)$ at
$\alpha = \beta = \delta = 1, \omega = 10^{-3}$, $a = \tau = 0.25$, $b = \tau = 0.75$.

ϕ	$ANALYTICAL$	$ABRDTM(a)$	$ABRDTM(b)$	$\lvert E - ABRDTM \rvert$
0.1	0.1097563	0.08989609	0.08477066	1.9860×10^{-2}
0.2	0.2084182	0.18855896	0.18343353	1.9859×10^{-2}
0.3	0.3049976	0.28513758	0.28001215	1.9860×10^{-2}
0.4	0.3985296	0.37866936	0.37354393	1.9860×10^{-2}
0.5	0.4880796	0.46822037	0.46309494	1.9859×10^{-2}
0.6	0.5727528	0.55289364	0.54776821	1.9859×10^{-2}
0.7	0.6517034	0.63184316	0.62671773	1.9860×10^{-2}
0.8	0.7241423	0.70428308	0.69915765	1.9859×10^{-2}
0.9	0.78934585	0.76948660	0.76436117	1.9859×10^{-2}
1.0	0.8466624952	0.82680324	0.82167780	1.9859×10^{-2}

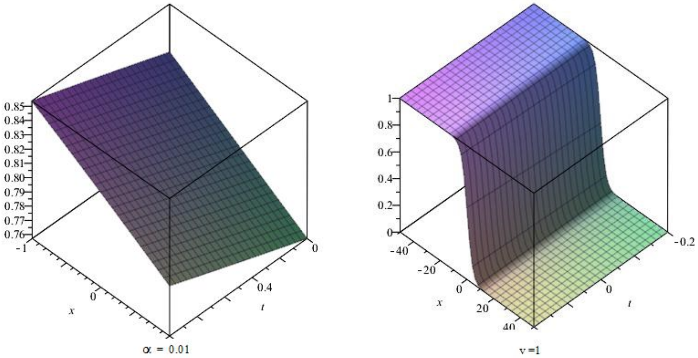
0.85
0.84
0.83
0.82
0.81
0.80
0.79
0.78
0.77
0.76
-1
0
x
0.4
t
0
α = 0.01
1
0.8
0.6
0.4
0.2
0
-40
-20
0
x
20
40
0
t
-0.2
v =1

Figure: 1

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