# Unsteady Flow of Micropolar Nanofluid over a Stratified Stretching Surface with Riga Plate.

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28th, May 2024



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- Unsteady Flow
- Micropolar Nanofluid
- Riga Plate



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The aim of this research is to scrutinize micropolar nanofluids with specific interest in the fluid behaviour over a stratified stretching surface with Riga plate under unsteady conditions.

To attain this will require the under-listed objectives;

i.Formulating a mathematical model which represents the flow scenario.

ii. Introducing some similarity variables, to transform the formulated boundary layer equations (nonlinear partial differential equations) into coupled ODEs with nonlinearities for the stratified and controlled regimes.
iii. Obtaining the numerical solutions for the derived ordinary differential equations.

**iv**. Investigating the effects of controlling parameters on the velocity, temperature and concentration phases of the flow.



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Hayat et al. (2017) examined the Brownian motion and thermophoresis aspects in nonlinear flow of micropolar nanoliquid. Stretching surface with linear velocity created the flow. Energy expression is modelled subject to consideration of thermal radiation phenomenon. Effect of Newtonian heating was found to enhance the temperature profile. Rafique et al. (2022) discussed the numerical analysis of the energy and mass transport behavior of microrotational flow via Riga plate, considering suction or injection and mixed convection. The thermal stratified parameters of nanofluid were captured using an interpretation of the well-known Keller box model, which helps us to determine the characteristic properties of the physical parameters. The fluid velocity was

enhanced by an increase in the modified Hartmann number



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$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
(1)  

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(\frac{\mu + k_1^*}{\rho}\right)\frac{\partial^2 u}{\partial y^2} + \left(\frac{k_1^*}{\rho}\right)\frac{\partial N^*}{\partial y} + g\left[\beta_t(T - T_\infty) + \beta_c(C - C_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right)e^{-1}$$
(2)  

$$u\frac{\partial N^*}{\partial x} + v\frac{\partial N^*}{\partial y} = \left(\frac{\gamma^*}{j^*\rho}\right)\frac{\partial^2 N^*}{\partial y^2} - \left(\frac{k_1^*}{j^*\rho}\right)\left(2N^* + \frac{\partial v}{\partial y}\right)$$
(3)  

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau \left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]$$
(4)  

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}$$
(5)



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$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left(\frac{\mu + k_1^*}{\rho}\right) \frac{\partial^2 u}{\partial y^2} + \left(\frac{k_1^*}{\rho}\right) \frac{\partial N^*}{\partial y} + g \left[\beta_t (T - T_\infty) + \beta_c (C - C_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right) \frac{\partial N_0}{\partial y} + g \left[\beta_t (T - T_\infty) + \beta_c (C - C_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right) \frac{\partial N_0}{\partial y} + g \left[\beta_t (T - T_\infty) + \beta_c (C - C_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right) \frac{\partial N_0}{\partial y} + g \left[\beta_t (T - T_\infty) + \beta_c (C - C_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right) \frac{\partial N_0}{\partial y} + g \left[\beta_t (T - T_\infty) + \beta_c (C - C_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right) \frac{\partial N_0}{\partial y} + g \left[\beta_t (T - T_\infty) + \beta_c (C - C_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right) \frac{\partial N_0}{\partial y} + g \left[\beta_t (T - T_\infty) + \beta_c (C - C_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right) \frac{\partial N_0}{\partial y} + g \left[\beta_t (T - T_\infty) + \beta_t (C - C_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right) \frac{\partial N_0}{\partial y} + g \left[\beta_t (T - T_\infty) + \beta_t (C - C_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right) \frac{\partial N_0}{\partial y} + g \left[\beta_t (T - T_\infty) + \beta_t (C - C_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right) \frac{\partial N_0}{\partial y} + g \left[\beta_t (T - T_\infty) + \beta_t (C - C_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right) \frac{\partial N_0}{\partial y} + g \left[\beta_t (T - T_\infty) + \beta_t (C - C_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right) \frac{\partial N_0}{\partial y} + g \left[\beta_t (T - T_\infty) + \beta_t (C - C_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right) \frac{\partial N_0}{\partial y} + g \left[\beta_t (T - T_\infty) + \beta_t (T - T_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right) \frac{\partial N_0}{\partial y} + g \left[\beta_t (T - T_\infty) + \beta_t (T - T_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right) \frac{\partial N_0}{\partial y} + g \left[\beta_t (T - T_\infty) + \beta_t (T - T_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right) \frac{\partial N_0}{\partial y} + g \left[\beta_t (T - T_\infty) + \beta_t (T - T_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right) \frac{\partial N_0}{\partial y} + g \left[\beta_t (T - T_\infty) + \beta_t (T - T_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right) \frac{\partial N_0}{\partial y} + g \left[\beta_t (T - T_\infty) + \beta_t (T - T_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right) \frac{\partial N_0}{\partial y} + g \left[\beta_t (T - T_\infty) + \beta_t (T - T_\infty)\right] + \left(\frac{\pi j_0 M_0}{8\rho}\right) \frac{\partial N_0}{\partial y} + \left(\frac{\pi j_0 M_0}{8\rho}\right)$$

$$\frac{\partial N^*}{\partial t} + u \frac{\partial N^*}{\partial x} + v \frac{\partial N^*}{\partial y} = \left(\frac{\gamma^*}{j^* \rho}\right) \frac{\partial^2 N^*}{\partial y^2} - \left(\frac{k_1^*}{j^* \rho}\right) \left(2N^* + \frac{\partial v}{\partial y}\right)$$
(8)

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[ D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right] + \frac{Q}{\rho C \rho} (T - T_\infty)$$
(9)

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2}$$
(10)



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$$u = u_0 e^{\frac{x}{2l}} f'(\eta), v = -\sqrt{\frac{vu_0}{2l}} \{f(\eta) + \eta f'(\eta)\} e^{\frac{x}{2l}}, \eta = y \sqrt{\frac{u_0}{2lv}} e^{\frac{x}{2l}}$$
(11)  
$$\theta(\eta) = \frac{T - T_0}{T_w - T_0}, \ \phi(\eta) = \frac{C - C_0}{C_w - C_0}, \ N^* = \left(\frac{u_0}{2lv}\right) e^{\frac{3x}{2l}} \sqrt{2lvu_0} h(\eta)$$



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Introducing the similarity transformation from equation (11) into equations (7) to (10) leads to (12) to (15)

$$(1+K)f'''+k'+\lambda\theta+8\phi+Me^{-m\eta}-A\left(f'+\frac{1}{2}\eta f''\right)-(f')^{2}+ff''=0 \quad (12)$$

$$(1+\frac{K}{2})h'' - K(2h+f'') - \frac{b}{2a}(3h+\eta h') - f'h + fh' = 0$$
(13)

$$\theta'' + p_r N b \phi' \theta' + p_r N_t \theta'^2 - \frac{3}{2} A p_r \theta - \frac{A}{2} \eta p_r \theta' - 2 p_r f' \theta + p_r f \theta' = 0$$
(14)

$$\phi^{\prime\prime} + \frac{N_t}{N_b}\theta^{\prime\prime} - \frac{3A}{2}Le\phi + \frac{A}{2}\eta \ Le\phi^{\prime} - 2Lef^{\prime}\phi - Le\theta^{\prime} = 0$$
(15)

With the transformed boundary conditions and Maple 18.0 Software, the results and graphical representations were obtained.



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The numerical simulation for the solution of equation (12) to (15) subject to the boundary condition (11) are graphically illustrated as follows:





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This research addressed stratified micropolar nanofluid flow over an exponentially stretchable Riga surface. The Runge Kutta was applied to obtain the results of the modeled flow equations. From the analysis: The velocity profile increased with an increase in magnetic parameter, the velocity and temperature profiles decreased while the concentration profile increased with an increase parameter.



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