

ICAWMSCS Seminar Series

International Conference and Advanced Workshop on Modelling and Simulation of Complex Systems

The Benefits and Risks of Social Media Use: An Age-Structured Mathematical Model with Optimal Control

Department of Mathematics
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The Benefits and Risks of Social Media Use

An Age-Structured Mathematical Model
with Optimal Control



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Motivation and modelling question



Unguided engagement among younger users



Potential harms also affect adults

Central question

Under what conditions can media engagement persist, and how can beneficial and harmful outcomes coexist?

Motivation: Valkenburg et al. (2022); Orben et al. (2024). Images are illustrative.

Related literature: evidence on social-media effects

Study	Contribution	Scope and remaining question
Valkenburg et al. (2022)	Synthesized 25 reviews. Associations with adolescent mental health were often weak or inconsistent.	Evidence synthesis does not supply a dynamical model of beneficial and harmful pathways.
Orben et al. (2024)	Reviewed developmental mechanisms involving social comparison, feedback and stress sensitivity.	Focuses on adolescent vulnerability rather than jointly modelling adult and underage trajectories.

Implication for the model

Age and the type of engagement matter. A single measure of time online may hide differences between benefits and harms.

The scope column describes differences from this framework, rather than claiming that the studies are defective.

Related literature: mathematical modelling

Study	Contribution	Scope and remaining question
Alemneh & Alemu (2021)	Developed a deterministic social-media addiction model with equilibrium analysis and optimal controls.	Addiction-centred formulation; it does not use the beneficial/harmful media and outcome structure presented here.
Ali et al. (2024)	Modelled addiction and depression, with threshold, stability, sensitivity and RK4 simulations.	Clinical harm is central. The study reports no generated or analysed empirical dataset for its model.

Contribution of this study

We combine age structure, separate beneficial and harmful pathways, qualitative analysis and semi-analytical verification.

Recent literature: interventions and within-person dynamics

Study	Contribution	Limitations and modelling relevance
Poli et al. (2025)	Reviewed risk/protective factors and interventions for problematic use. Some interventions showed promise.	Few, varied studies and limited intervention evidence restrict conclusions. Supports cautious policy assessment.
Shawcroft & Cingel (2026)	Followed 97 US adolescents. Overall associations were limited. Use and unwanted explicit content related to less stable sense of purpose.	Small sample and self-reported use limit inference. Highlights specific experiences and longitudinal assessment.

Implication for this research

Distinguishing engagement pathways is useful, but quantitative intervention effects require stronger empirical calibration.

Research aim, objective, question and methods

Aim

To understand the benefits and risks of social-media use and identify interventions that reduce harmful engagement.

Objective

To develop and analyse an age-structured mathematical model and evaluate optimal-control strategies.

Research question

Under what conditions do beneficial and harmful pathways persist, and how effectively can guidance, moderation and digital literacy reduce harm?

Methods

Model formulation; positivity, boundedness and existence analysis; equilibrium, threshold, stability and bifurcation analysis; simulations and sensitivity analysis; survey assessment; DTM-RK4 comparison; optimal-control analysis.

Baseline compartments and assumptions

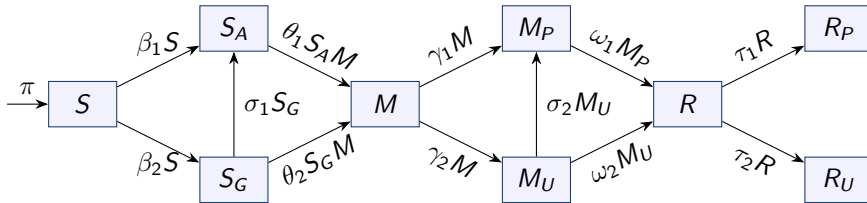
Symbol	Meaning
S	General susceptible class
S_A, S_G	Adult and underage classes
M	Unclassified media pool
M_P, M_U	Beneficial and harmful pathways
R	Emerging outcomes
R_P, R_U	Positive and harmful outcomes

The baseline has nine states. Individuals aged 0–5 are excluded from independent use.

Assumptions

Underage users are 6–17 years; adults are 18+.
Rates remain constant within each scenario.
Interaction is homogeneous within compartments. Underage users can mature into the adult class.

Baseline transition diagram



Exit terms

Natural exit μ acts on every state; additional exits δ_1 and δ_2 act on M_U and R_U .

Transitions follow the baseline equations; exit terms are listed separately.

Baseline equations: age structure and media generation

$$\begin{aligned}\dot{S} &= \pi - (\beta_1 + \beta_2 + \mu)S, \\ \dot{S}_A &= \beta_1 S + \sigma_1 S_G - \theta_1 S_A M - \mu S_A, \\ \dot{S}_G &= \beta_2 S - (\sigma_1 + \mu)S_G - \theta_2 S_G M, \\ \dot{M} &= [\theta_1 S_A + \theta_2 S_G - (\gamma_1 + \gamma_2 + \mu)]M.\end{aligned}$$

Nonlinearity

The products $S_A M$ and $S_G M$ couple age-specific exposure to the media pool.

Baseline equations: beneficial and harmful outcomes

$$\dot{M}_P = \gamma_1 M + \sigma_2 M_U - (\omega_1 + \mu) M_P,$$

$$\dot{M}_U = \gamma_2 M - (\sigma_2 + \omega_2 + \delta_1 + \mu) M_U,$$

$$\dot{R} = \omega_1 M_P + \omega_2 M_U - (\tau_1 + \tau_2 + \mu) R,$$

$$\dot{R}_P = \tau_1 R - \mu R_P,$$

$$\dot{R}_U = \tau_2 R - (\delta_2 + \mu) R_U.$$

All initial states are non-negative. Beneficial and harmful pathways both feed the common intermediate outcome state R .

Well-posedness, positivity and boundedness

Existence and non-negativity

The polynomial vector field is locally Lipschitz, giving a unique local solution. At each non-negative boundary, the corresponding derivative is non-negative:

$$\dot{S}|_{S=0} = \pi, \quad \dot{M}_U|_{M_U=0} = \gamma_2 M \geq 0.$$

Thus non-negative initial states remain non-negative.

If $W(0) \leq \pi/\mu$, the non-negative region $W \leq \pi/\mu$ is positively invariant. For $N_h = S + S_A + S_G$, the same comparison bound holds.

W is a mathematical aggregate on compatible scales, not a literal population count.

Complete-system bound

For the sum W of the nine states,

$$\begin{aligned}\dot{W} &= \pi - \mu W - \delta_1 M_U - \delta_2 R_U \\ &\leq \pi - \mu W.\end{aligned}$$

Hence $W(t) \leq \max\{W(0), \pi/\mu\}$. These bounds prevent finite-time blow-up and ensure global existence.

Media-free equilibrium and persistence threshold

At $E_0 = (S^0, S_A^0, S_G^0, 0, 0, 0, 0, 0, 0)$,

$$S^0 = \frac{\pi}{\beta_1 + \beta_2 + \mu}, \quad S_G^0 = \frac{\beta_2 S^0}{\sigma_1 + \mu}, \quad S_A^0 = \frac{\beta_1 S^0 + \sigma_1 S_G^0}{\mu}.$$

Near E_0 , the media equation is

$$\dot{M} = [\theta_1 S_A^0 + \theta_2 S_G^0 - (\gamma_1 + \gamma_2 + \mu)]M.$$

Exposure threshold

$$\mathcal{R}_0 = \frac{\theta_1 S_A^0 + \theta_2 S_G^0}{\gamma_1 + \gamma_2 + \mu}$$

Media generation relative to exit from the unclassified pool governs persistence.

Substitution yields

$$\mathcal{R}_0 = \frac{\pi[\theta_1\{\beta_1(\sigma_1 + \mu) + \sigma_1\beta_2\} + \mu\theta_2\beta_2]}{\mu(\beta_1 + \beta_2 + \mu)(\sigma_1 + \mu)(\gamma_1 + \gamma_2 + \mu)}.$$

Local stability of the media-free equilibrium

Threshold-dependent eigenvalue

$$\lambda_M = (\gamma_1 + \gamma_2 + \mu)(\mathcal{R}_0 - 1).$$

The remaining eight eigenvalues are negative for positive parameters.

$$\mathcal{R}_0 < 1$$

All eigenvalues have negative real parts.
 E_0 is locally asymptotically stable.

$$\mathcal{R}_0 > 1$$

The media eigenvalue is positive. E_0 is unstable.

At $\mathcal{R}_0 = 1$, linearization is inconclusive; the bifurcation analysis addresses that case.

Persistent equilibrium: reduction to one equation

For $M^* > 0$, let $S^* = \pi/(\beta_1 + \beta_2 + \mu)$. Then

$$S_G^*(m) = \frac{\beta_2 S^*}{\sigma_1 + \mu + \theta_2 m}, \quad S_A^*(m) = \frac{\beta_1 S^* + \sigma_1 S_G^*(m)}{\mu + \theta_1 m}.$$

The media equilibrium requires

$$F(m) := \theta_1 S_A^*(m) + \theta_2 S_G^*(m) = \gamma_1 + \gamma_2 + \mu.$$

Existence and uniqueness

For positive parameters, $F(m)$ decreases with m , tends to zero, and $F(0) = (\gamma_1 + \gamma_2 + \mu)\mathcal{R}_0$. Thus there is exactly one positive solution when $\mathcal{R}_0 > 1$.

Once the positive media equilibrium is determined, downstream states follow sequentially.

Persistent equilibrium: downstream components

$$M_U^* = \frac{\gamma_2 M^*}{\sigma_2 + \omega_2 + \delta_1 + \mu},$$

$$R^* = \frac{\omega_1 M_P^* + \omega_2 M_U^*}{\tau_1 + \tau_2 + \mu},$$

$$R_U^* = \frac{\tau_2 R^*}{\delta_2 + \mu}.$$

$$M_P^* = \frac{\gamma_1 M^* + \sigma_2 M_U^*}{\omega_1 + \mu},$$

$$R_P^* = \frac{\tau_1 R^*}{\mu},$$

Coexistence

With $M^* > 0$ and positive pathway parameters, both beneficial and harmful media states and outcomes can persist.

Persistent-equilibrium stability: Jacobian and criterion

The core Jacobian for (S, S_A, S_G, M) is

$$J_c = \begin{pmatrix} -a_1 & 0 & 0 & 0 \\ \beta_1 & -a_2 & \sigma_1 & -\theta_1 S_A^* \\ \beta_2 & 0 & -a_3 & -\theta_2 S_G^* \\ 0 & \theta_1 M^* & \theta_2 M^* & 0 \end{pmatrix},$$

$$a_1 = \beta_1 + \beta_2 + \mu, \quad a_2 = \mu + \theta_1 M^*, \quad a_3 = \sigma_1 + \mu + \theta_2 M^*.$$

Routh–Hurwitz test

For $P_c(\lambda) = \lambda^4 + A_1\lambda^3 + A_2\lambda^2 + A_3\lambda + A_4$, require

$$A_i > 0, \quad A_1 A_2 > A_3, \quad A_1 A_2 A_3 > A_3^2 + A_1^2 A_4.$$

The five downstream eigenvalues are negative. These conditions give local asymptotic stability.

A sufficient global condition

With $S^0 = \pi/(\beta_1 + \beta_2 + \mu)$,

$$\frac{\max\{\theta_1, \theta_2\}(\beta_1 + \beta_2)S^0}{\mu(\gamma_1 + \gamma_2 + \mu)} < 1.$$

Comparison bounds imply media decay and convergence to E_0 . This condition is stronger than the local threshold test.

Forward bifurcation

Scaling exposure rates by ϕ gives a critical value where $\mathcal{R}_0(\phi) = 1$. The system exhibits a forward transcritical bifurcation: a positive equilibrium branch emerges above the threshold.

Baseline simulation specification

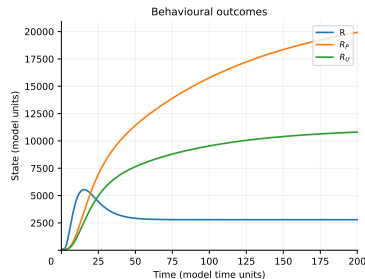
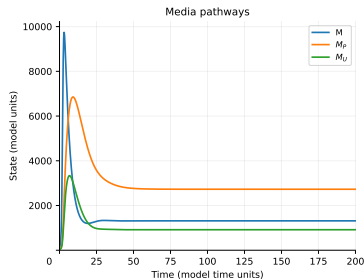
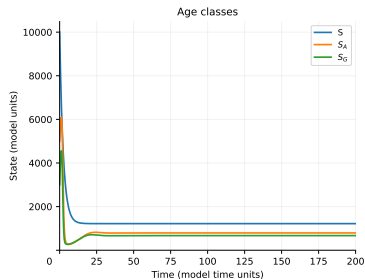
Parameter group	Values
π, μ	500, 0.01
β_1, β_2	0.15, 0.25
σ_1, σ_2	0.05, 0.10
γ_1, γ_2	0.20, 0.15
θ_1, θ_2	0.0002, 0.0003
ω_1, ω_2	0.12, 0.10
τ_1, τ_2	0.08, 0.06
δ_1, δ_2	0.005, 0.005

Initial state

(10000, 5000, 3000, 100, 50, 50, 100, 55, 45) in the ordering $(S, S_A, S_G, M, M_P, M_U, R, R_P, R_U)$.

Baseline threshold calculated from these values: $\mathcal{R}_0 \approx 28.51$. Rates are illustrative model assumptions, not fully fitted estimates.

Baseline dynamics and coexistence of outcomes



Interpretation

Media engagement persists while both positive and harmful outcome states accumulate. These trajectories describe the stated model scenario.

Numerical solution of the baseline equations using the stated parameters; horizon 0–200 model time units.

Sensitivity: exposure and content pathways

Since $\mathcal{R}_0 = (\theta_1 S_A^0 + \theta_2 S_G^0)/(\gamma_1 + \gamma_2 + \mu)$,

$$\frac{\partial \mathcal{R}_0}{\partial \theta_i} > 0, \quad \frac{\partial \mathcal{R}_0}{\partial \gamma_j} < 0.$$

Endpoint sensitivity is examined through joint changes in γ_1, γ_2 at $t = 200$:

Scenario	M_P	M_U	R_P	R_U
Low	2694.1514	905.7923	19655.0950	10673.3420
Baseline	2727.4200	916.9774	19927.6685	10809.0840
High	2710.0914	911.1514	19785.6932	10733.9769

Endpoint outcomes need not vary monotonically with the content-transition rates.

Endpoint scenario values at $t = 200$. The low/high parameter settings require specification for full reproducibility.

Empirical assessment of the dual pathways

Reported impact ratings

Among 142 valid ratings, 73.9% reported large or extremely large positive impact; 46.5% reported corresponding negative impact.

Association

Positive and negative ratings had weak association: $\rho = 0.117$, $p = 0.167$. This is compatible with separate pathways but does not prove their independence.

Scope of evidence

The survey assesses conceptual and qualitative structural correspondence. Longitudinal observations are needed to identify transition rates and evaluate time trajectories.

Total survey responses: 143; valid age classifications: 133; valid impact ratings: 142.

Age-specific extension for semi-analytical analysis

Baseline model

M_U is one aggregate harmful class. The exposure threshold and equilibrium conditions apply to this nine-state formulation.

The additional harmful pathway introduces γ_3 , age-specific redirection and exit rates. Setting all three controls to zero yields the uncontrolled system.

Extended model

The extension separates harmful engagement into M_{UA} and M_{UG} , giving ten states. A DTM approximation describes its short-term uncontrolled dynamics.

DTM construction and recurrence

Following Ayaz (2004), define $X_k = x^{(k)}(0)/k!$ and reconstruct

$$x(t) \approx \sum_{k=0}^{10} X_k t^k.$$

Derivative rule

$$\mathcal{D}[x']_k = (k+1)X_{k+1}.$$

Product rule

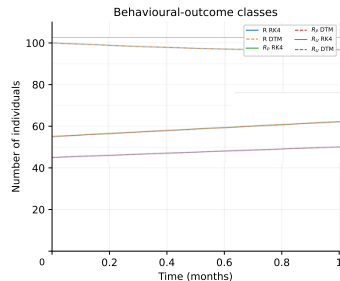
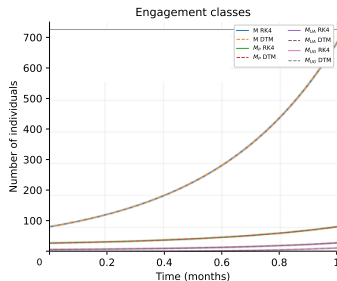
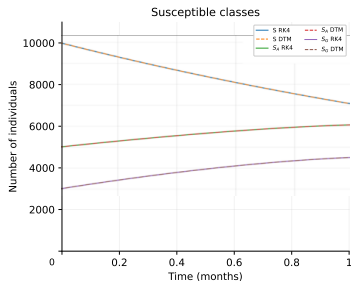
$$\mathcal{D}[xy]_k = \sum_{\ell=0}^k X_{\ell} Y_{k-\ell}.$$

For adult susceptible coefficients a_k , a representative recurrence is

$$(k+1)a_{k+1} = \beta_1 s_k + \sigma_1 g_k - \mu a_k - \theta_1 \sum_{\ell=0}^k a_{\ell} m_{k-\ell}.$$

Initial values supply the zeroth coefficients. Iteration determines higher coefficients through order 10 for the uncontrolled ten-state extension.

DTM–RK4 trajectories and interpretation



Dynamics: S declines while S_A , S_G increase. Engagement states grow, especially M . R declines slightly, while positive and harmful outcomes increase.

Agreement: Solid RK4 and dashed DTM curves nearly overlap. At one month, the largest reported difference is 0.397 in M (about 0.058%).

Tenth-order DTM; RK4 $h = 0.01$; $0 \leq t \leq 1$ month. Agreement supports short-term approximation, not empirical validation.

Accuracy of the DTM approximation at one month

State	DTM	RK4	Absolute difference
S	7046.685123	7046.685123	6.06×10^{-9}
S_A	6065.896141	6066.072328	0.176
S_G	4513.366572	4513.610194	0.244
M	690.238248	689.841738	0.397
M_P	102.457507	102.444321	0.0132
M_{UA}	51.159756	51.153853	0.00590
M_{UG}	35.044904	35.040950	0.00395

Selected numerical comparisons at one month; agreement between approximations does not establish predictive validation.

Intervention strategies and optimal control

The intervention extension adds guidance u_1 , moderation u_2 and digital literacy u_3 .

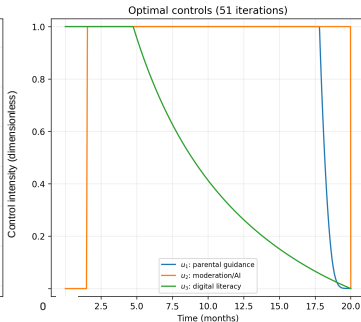
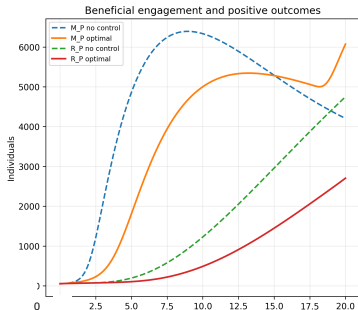
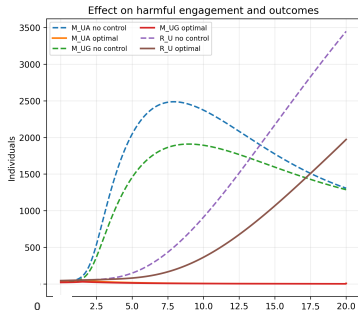
$$J = \int_0^T \left[A_1 M_{UA} + A_2 M_{UG} + A_3 R_U + \sum_{i=1}^3 \frac{B_i}{2} u_i^2 \right] dt.$$

Scenario result

The combined strategy reduces cumulative harmful adult and underage engagement by 99.31% and 99.40%, and harmful outcomes by 49.96%. These are model scenarios, not measured policy effects.

Intervention horizon: 0–20 months. Reductions refer to cumulative model quantities.

Optimal-control trajectories and interpretation



State trajectories: Combined controls suppress M_{UA} , M_{UG} and lower R_U . Beneficial engagement recovers later, although R_P is lower than in the uncontrolled scenario.

Control profiles: Guidance remains high until late in the horizon. Moderation rises early and stays high. Digital literacy starts high and then declines.

Horizon: 0–20 months. Left/middle: dashed uncontrolled, solid controlled. Right: control intensities. Reported percentage reductions are cumulative, not endpoint reductions.

State axes: individuals. Control intensity: dimensionless (0-1).

Conclusions and implications

Main finding

Social-media engagement can sustain both beneficial and harmful pathways. Age, engagement type and outcomes therefore matter when assessing its effects.

Mathematical and computational evidence

The model has non-negative, bounded solutions and a threshold governing media persistence. Simulations illustrate coexisting benefits and harms. DTM–RK4 agreement supports the short-term approximation, while combined controls substantially reduce harmful engagement in the intervention scenarios.

Implications and next step

The findings motivate coordinated parental guidance, platform moderation and digital literacy. Longitudinal data and parameter calibration are needed to assess the magnitude of these effects in practice.

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Thank you

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